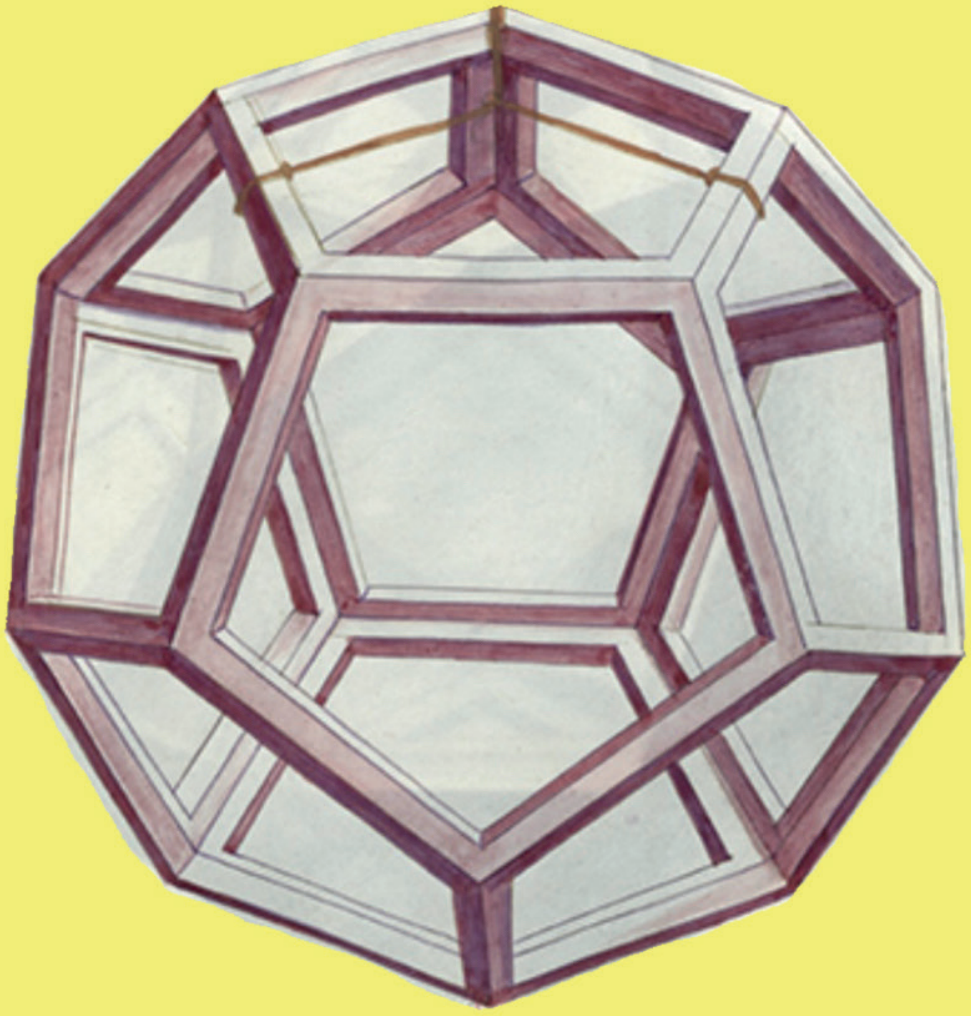
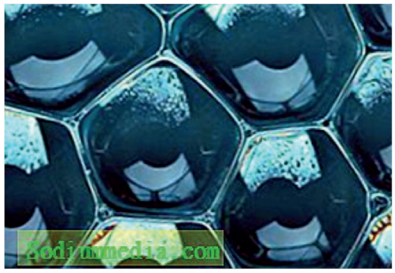
Jacopo Fo

Pythagoras and the Taoists

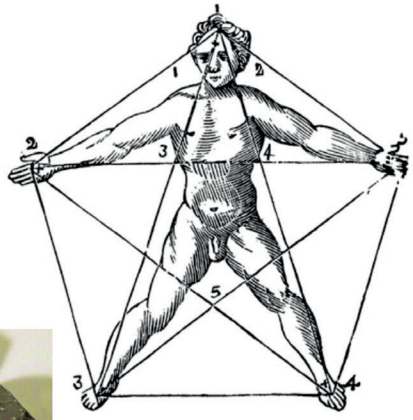


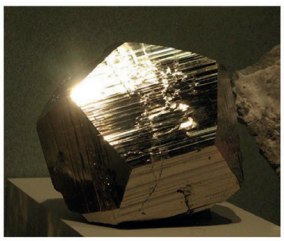
An archaeology of arithmetic and geometry
Short version













The forms of nature amazed the ancient philosophers

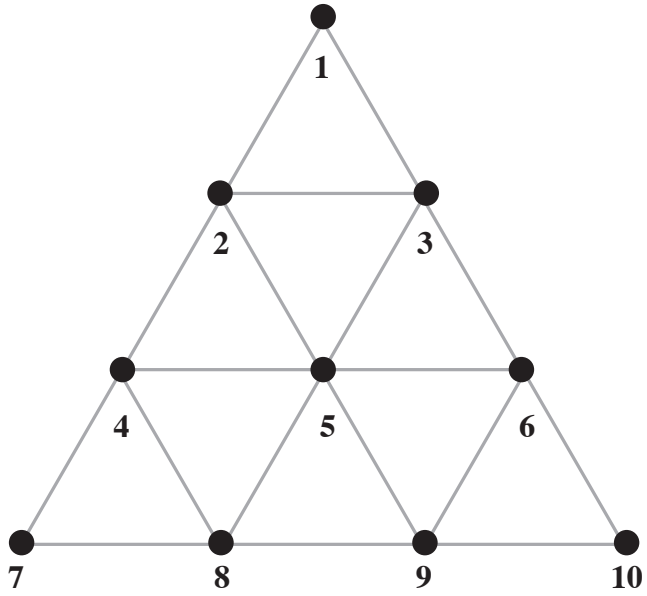
Human beings saw that nature often took on triangular, hexagonal, pentagonal forms, which we find in stones, crystals, starfish, flowers, leaves, fruits, and even in the shape of our body.

They also saw that the 'curds' in cauliflowers become increasingly smaller. They imagined that there was a sense to all this and created a fractal map of the relationship between shapes and numbers. Pythagoras named this map Tetractys.

We know today that these forms permeate the whole universe, from muscular fibres to some molecules, from Saturn's rings to snowflakes.

The discoveries of Pythagoras regarding the properties of numbers, the geometry and harmony of sounds are an ancient example of scientific reasoning.

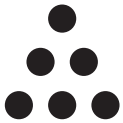
Only a few mathematicians, however, were interested in becoming familiar with the geometric and arithmetic model that he had elaborated starting from the alchemic theories of the Babylonians. Pythagoras was convinced that the figure of the Tetractys contained the key to understanding reality. Indeed, if we observe it carefully, many things can be discovered.



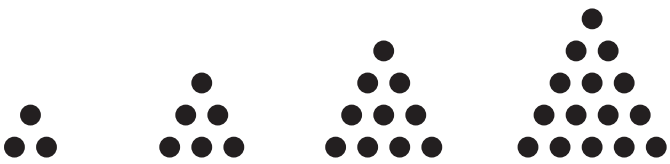
At school we learn that Pythagoras discovered triangular numbers, in other words those numbers which, when drawn as dots, can form a triangle (3, 6, 9…) and that he also discovered square numbers (4, 8, 16…).

For example, with 9 small stones I can form a square:

and with 6 stones a triangle:



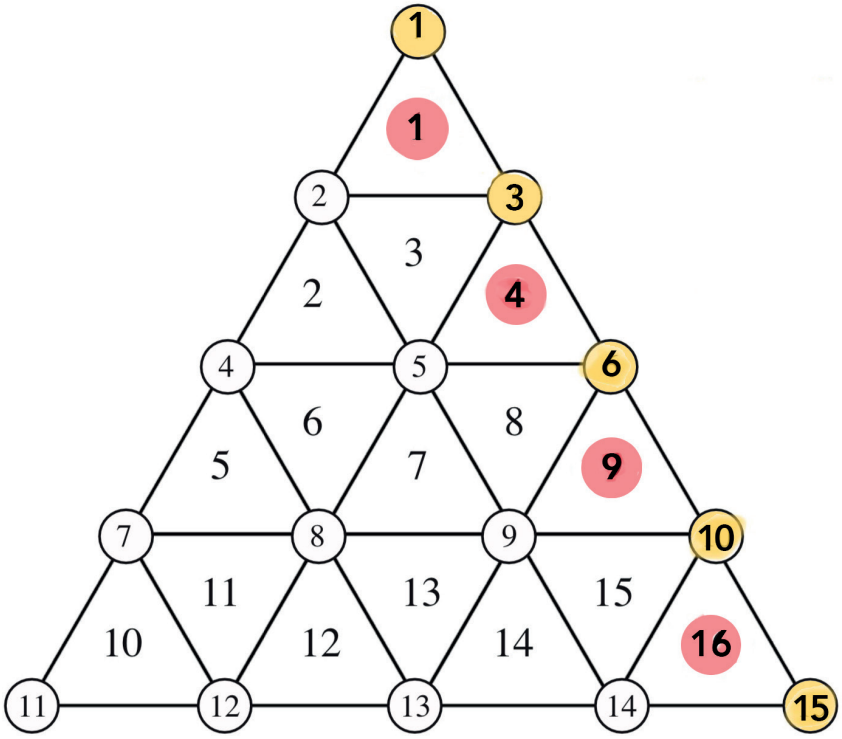
Triangular number series



Square number series

We are not taught, though, that these numbers emerge when looking at the Tetractys: the sides of the triangles that make up the right side are all triangular numbers, the areas along the same side are square numbers. As we will see later, however, this is just one of the properties of the Tetractys.

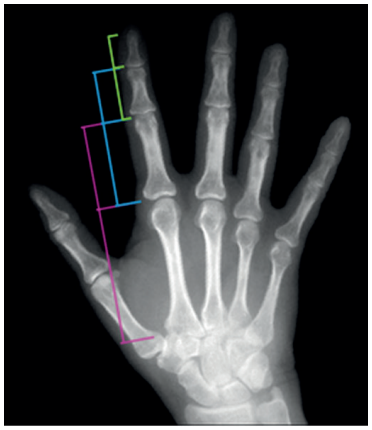
When Pythagoras said: "All is number", maybe he was saying that numbers give everything a shape.

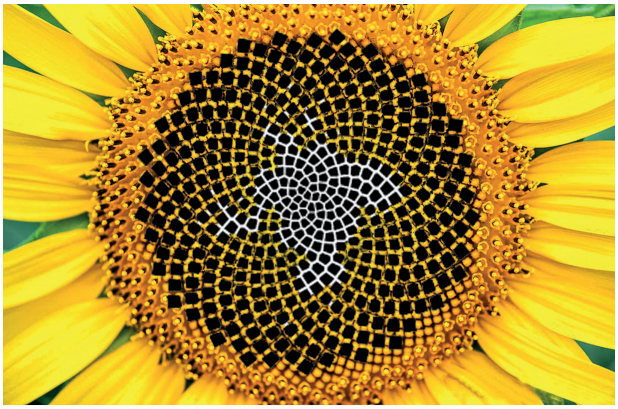


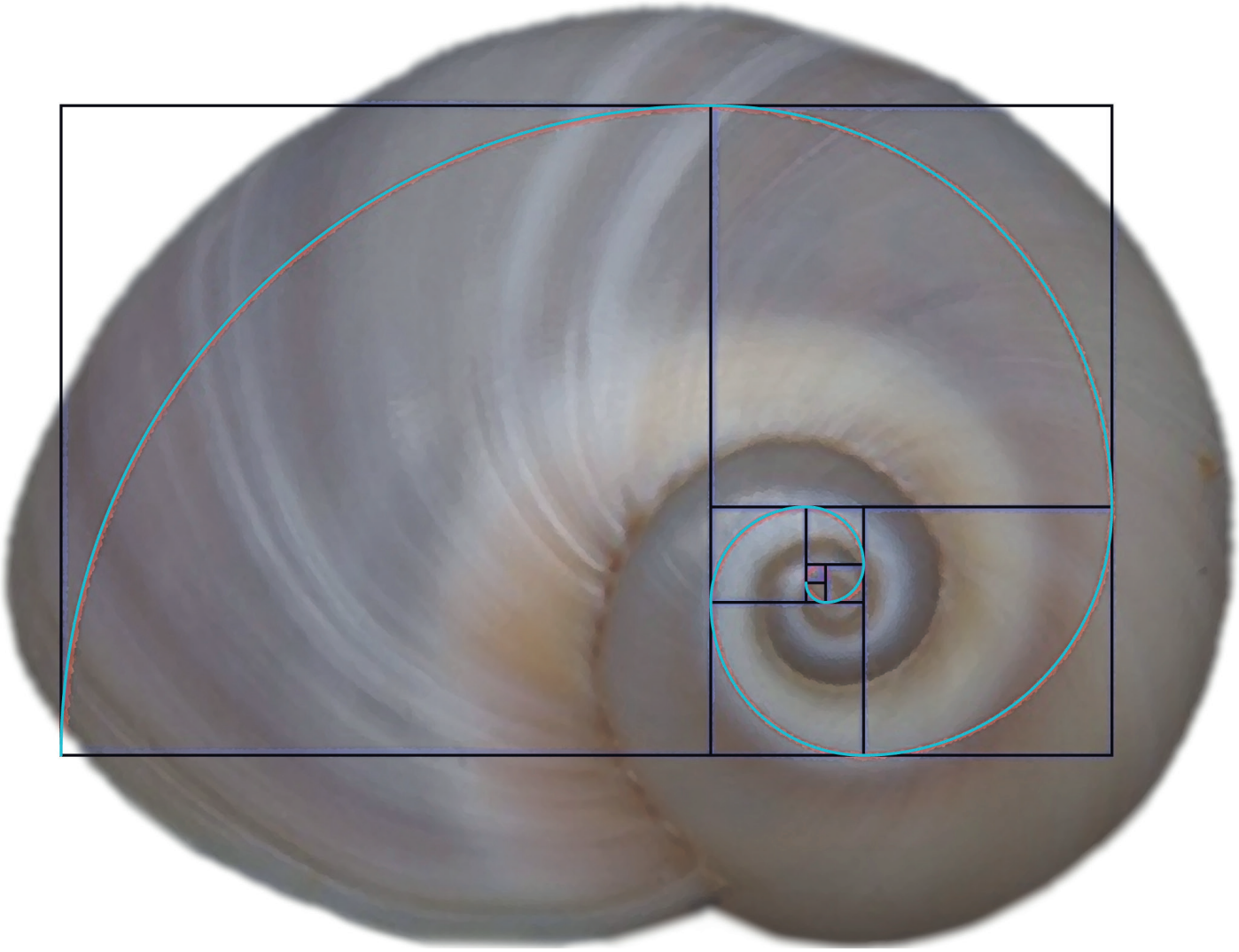
Triangular numbers	Square numbers
1	
3	1
6	4
10	9
15	16
21	25
28	36
36	49
45	64
55	81
66	100

This diagram contains the triangular and square numbers.

Following a curious procedure, the ancients came to discover the golden ratio centuries and centuries before Fibonacci described it in mathematical terms.







Is there a relationship between modern science and ancient arithmetic geometry?

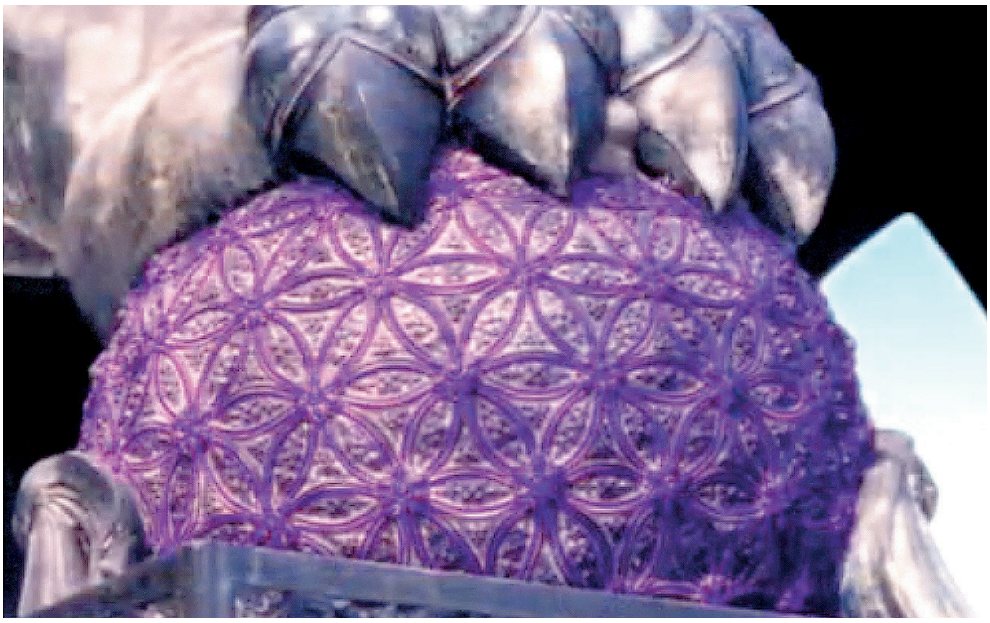
Kepler was one of the first to scientifically challenge the idea that the Earth was at the centre of the universe. He had to work hard, though, to save his mother Katharina from a witchcraft trial resulting from accusations that she had used a magic filter to poison a noblewoman.

The situation was complicated by the fact that Katharina was extremely combative and a times truly impolite. Kepler could not convince her to behave in a subdued manner.

The other element the prosecution had was that one of Katharina's aunts had been burned alive as a witch about ten years earlier. Then there was the fact that Kepler's mother was a herbalist who sold medicines and filters, a shaman who was probably familiar with ancient alchemy. Could some kind of scientific idea have come to Kepler as a result of what his mother had taught him?

In any case we know that his first work on the admirable proportions of the heavenly bodies and another work, Harmonices mundi, were permeated by Pythagoras's ideas on the proportions between numbers and geometric forms and Plato's verifications on perfect solids.

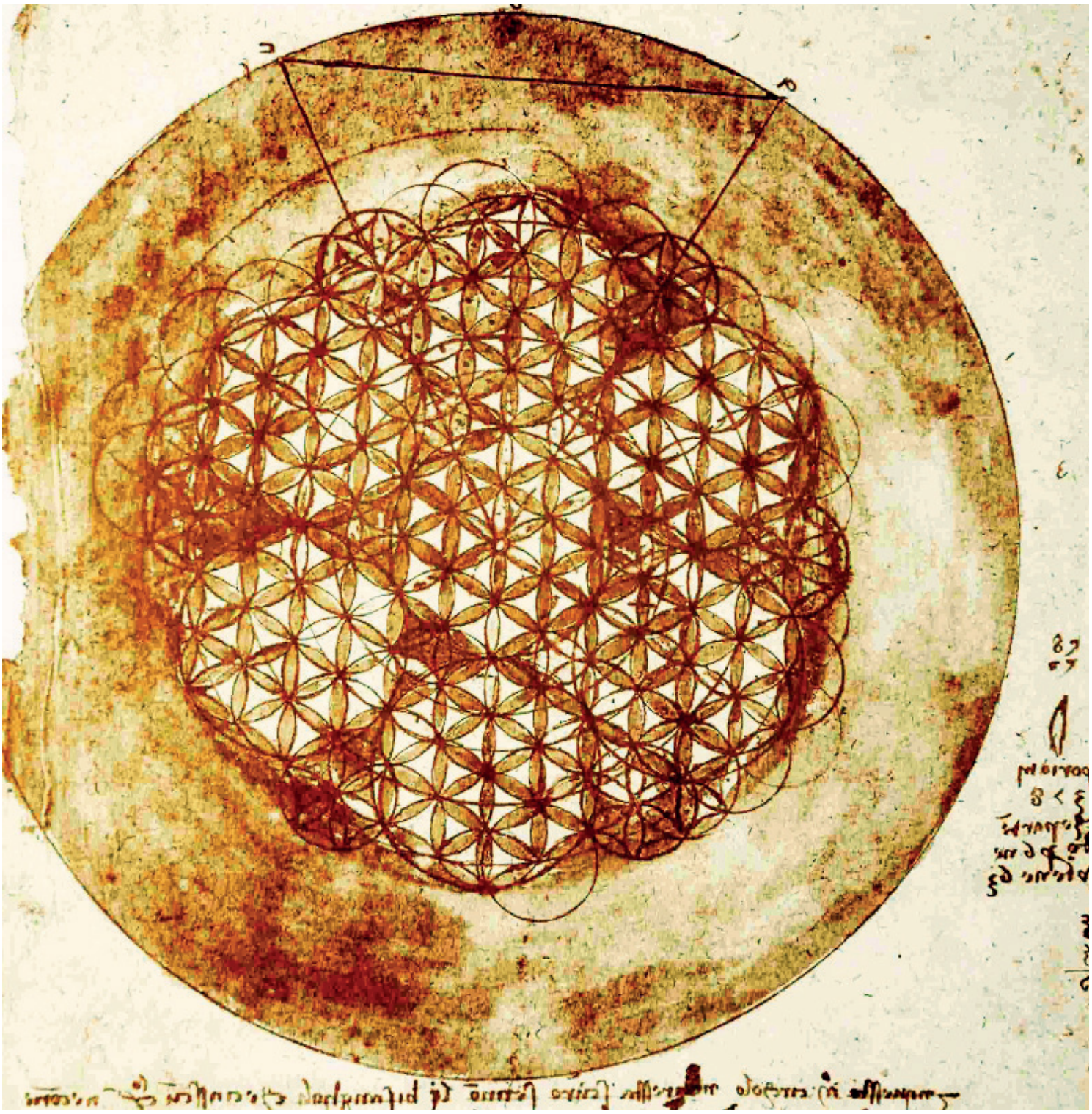
Everyone praises Galileo's discoveries, but few are interested in understanding how he developed them, starting from the teachings of his father, a gifted alchemist who had practiced the art of astrology, which in those days meant knowing Pythagorean and Hebrew arithmetic.



Fu Dog, the creature that guards the Forbidden City in Peking.

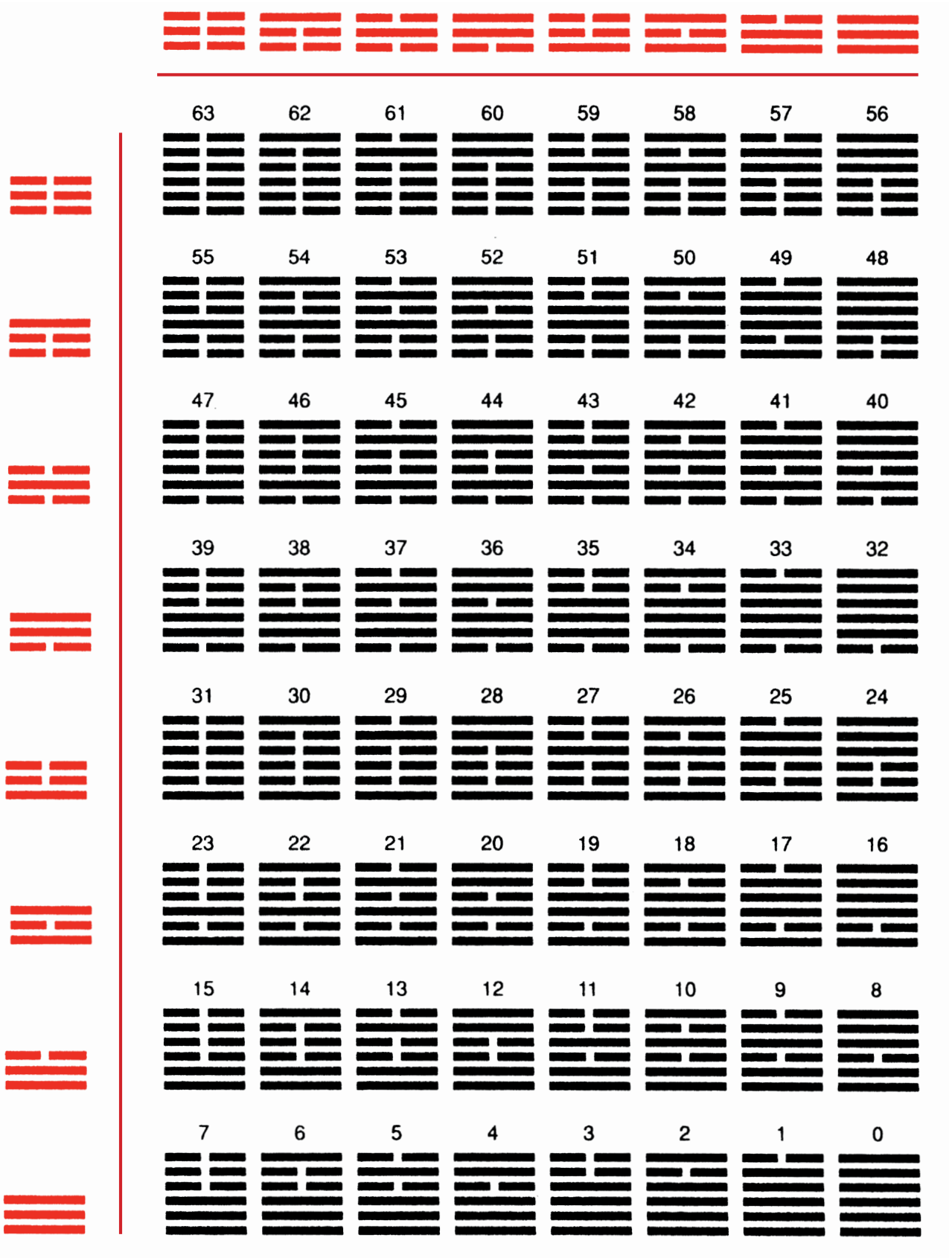
Few reflect on the fact that Leonardo da Vinci, when studying the laws of nature, dedicated a lot of time to understanding the meaning of the Pythagorean system based on the Pythagorean grid of triangles.

The fact that we find the reproduction of what is called the Flower of Life, an extended version of the Pythagorean Tetractys, among his drawings is proof of this; this drawing can be found in different cultures from Egypt to China.



Drawing by Leonardo da Vinci: the grid of equilateral triangles generates the Flower of Life.

Leibniz, the inventor of binary numbering, was very upset when a monk friend, returning from China, showed him the table of the 64 hexagrams (I King or I Ching) that illustrate the Taoist model of the world using symbols that are numbers expressed in binary language. The Chinese had discovered binary numbering at least two thousand years before Leibniz.



Even Newton is decreed to be a great innovator of science even though it is believed that he was slightly mad. It is a pity, though, that he decided to burn a whole trunk of alchemic notes because fearful that after his death they would be used to belittle him.

Modern mathematicians take no great interest in those few alchemic texts of Newton that were saved.

The Nobel prize winner Niels Bohr, considered one of the fathers of quantum mechanics, journeyed to China in 1937 and was greatly struck by the correspondences between the discoveries of modern physics and the Taoist model based on the complementarity of two opposing poles.

The physicist Fritjof Capra, in his wonderful book "The Tao of Physics", demonstrated that the fundamental laws of modern physics had already been used by the ancient Greek and Chinese philosophers.

I will stop here, even though I could continue at length on the search for the connections between scientific discoveries and the knowledge of an ancient model of creation based on arithmetic and elementary geometry.

Researchers still run away from this topic because commonly considered as a legacy of charlatan magic and irrational mystics. Even mystics, however, did not have any interest in understanding the rational, and in some way scientific, part of primitive alchemic thought.

The aspect that initially fascinated me and led me to go further into the question was the discovery that the model of the world as imagined by Pythagoras overlaps perfectly with the world of Taoism; as we will see, it is the same manner of observing reality and reasoning that identifies the same series of numbers and geometric shapes, elementary and "important", defined at times as "perfect" and which describes the same interrelations between them.

We find this model in more or less fragmented forms in very different and very distant cultures, from the Sumerians to the Jews, to the Indians, from the Egyptians to the Native Americans, to the Dogons of black Africa.

This similarity cannot depend on contacts that were certainly very sporadic, above all among philosophers… I believe that the correlation depends rather on the fact that this model is based on the discovery of a series of relationships that exist in nature; simple and evident aspects, relations between numbers and elementary geometric forms, shapes of rocks and flowers and aspects of the human body. The ancient philosophers asked themselves apparently stupid questions: why do insects have 6 or 8 legs and almost all mammals never more than 4? Does this reality hide something?

There is nothing mystical in their manner of observing the world, there are no transcendental visions or dogmatic revelations received directly from God.

I ask myself why, if dealing with the concrete aspects of reality, there is little interest in the matter and not even historians wanted to take a deeper look.

It would be an interesting topic, at least for research on the archaeology of science.

The obstacle lies in the fact that this model is only expressed with diagrams appearing here and there, without an indication of the connections between them. In addition, all the reasoning is based on an elementary manner of thinking; so elementary in fact that it seems to be one step above stupidity.

The idea of creating a map of quantitative evidence that recurs in reality is outside the contemporary way of reasoning: it seems to be a sterile exercise aimed at looking for coincidences where none exist. After all, they had nothing better to do because they did not have the technology that was required to explore reality in a more rational manner.

We are facing a system of relationships that can only be seen if observed together and if we are willing to follow an unusual thread of logic... But at this point quantum mechanics should have made us ready to evaluate ideas that at first glance appear strange.

We can certainly perceive an unusual way of seeing numbers and geometric characteristics.

For example, this model starts from the division of numbers not only into even and odd, but also into 3 fundamental categories: divisible by 3, even and not divisible by 3, odd and not divisible by 3.

odd numbers not divisible by 3	even numbers not divisible by 3	numbers divisible by 3
1	2	3
5	4	6
7	8	9
11	10	12
13	14	15
17	16	18
19	20	21

Division of the numbers into 3 categories.

I spoke with some maths teachers who told me that it is an irrelevant characteristic. The ancient sages classified about twenty irrelevant facts and demonstrated (from their point of view) that these aspects are closely connected and give life to a system, a series of natural quantitative relationships between numbers and forms.

They also thought that by discovering these relationships it would be possible to understand the ***natural quantum fractal laws*** that generate the world. But the concept of natural quantum laws already lies outside the modern model of reality. And even the concept of fractal, though known, is generally not considered a central element to the evolution of the universe.

In order to understand the ancient diagrams, you have to make your mind do a flip.

To explain myself I can refer to the Nazca, the Peruvian natives who dedicated

enormous resources to creating colossal drawings scraped on the desert crust. For decades archaeologists tried – without managing – to understand what they represented, until Maria Reich discovered that the drawings were the image of the darkest areas of the sky.

The Nazca, instead of drawing the lit points, the stars and the planets, drew the contours of the darkest areas, the areas where stars could not be seen. These drawings included lines that indicated the movements made by these black marks in the sky during the year. The drawings were an immense system that recorded the movements of the constellations seen as a photo negative.

Before continuing, maybe it is best to see the bases of this "flipped" reasoning.

We therefore begin with a series of geometric observations: from the discovery of strange optic effects and other strange properties of the forms that led to the ancients' passion for arithmetic.

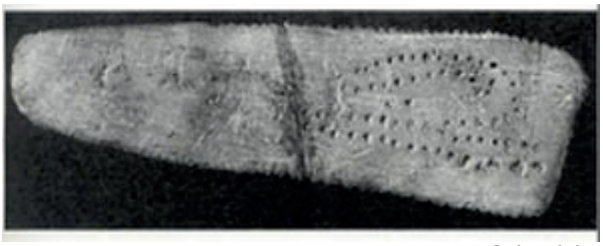
The discovery of Divine Harmony

The archaeology of thought: how the ancients, while building huts with branches and decorating vases, discovered stars hidden in pentagons that are hidden in cubes, hidden in hexagons, hidden in triangles.

One day a woman realized that when the moon was not in the sky she lost blood from her body.

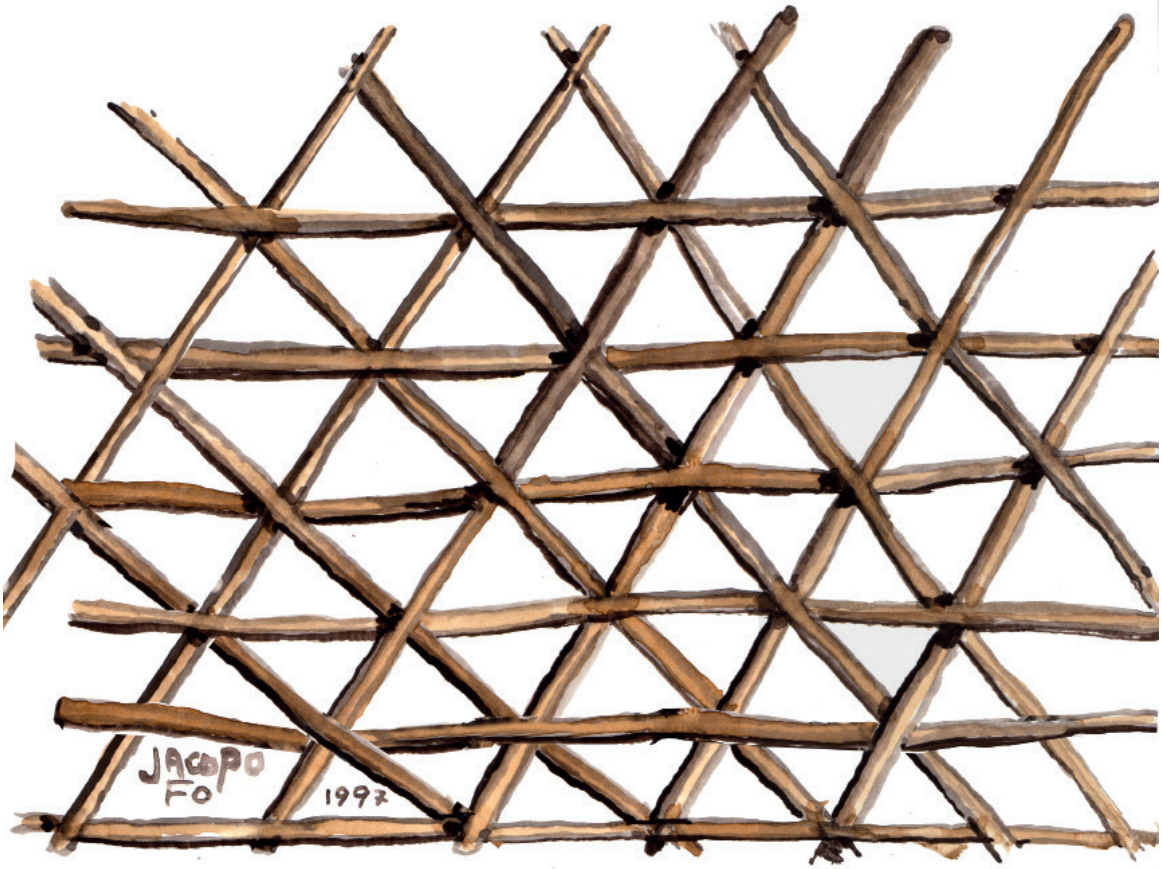
She thought, therefore, that there was a kind of correspondence between the moon and menstruation. She took a stone and drew a snake made up of 28 small holes. When the moon appeared in the sky like a small sickle, she placed a small stick into the first hole. Every day, she moved the stick into the next hole.

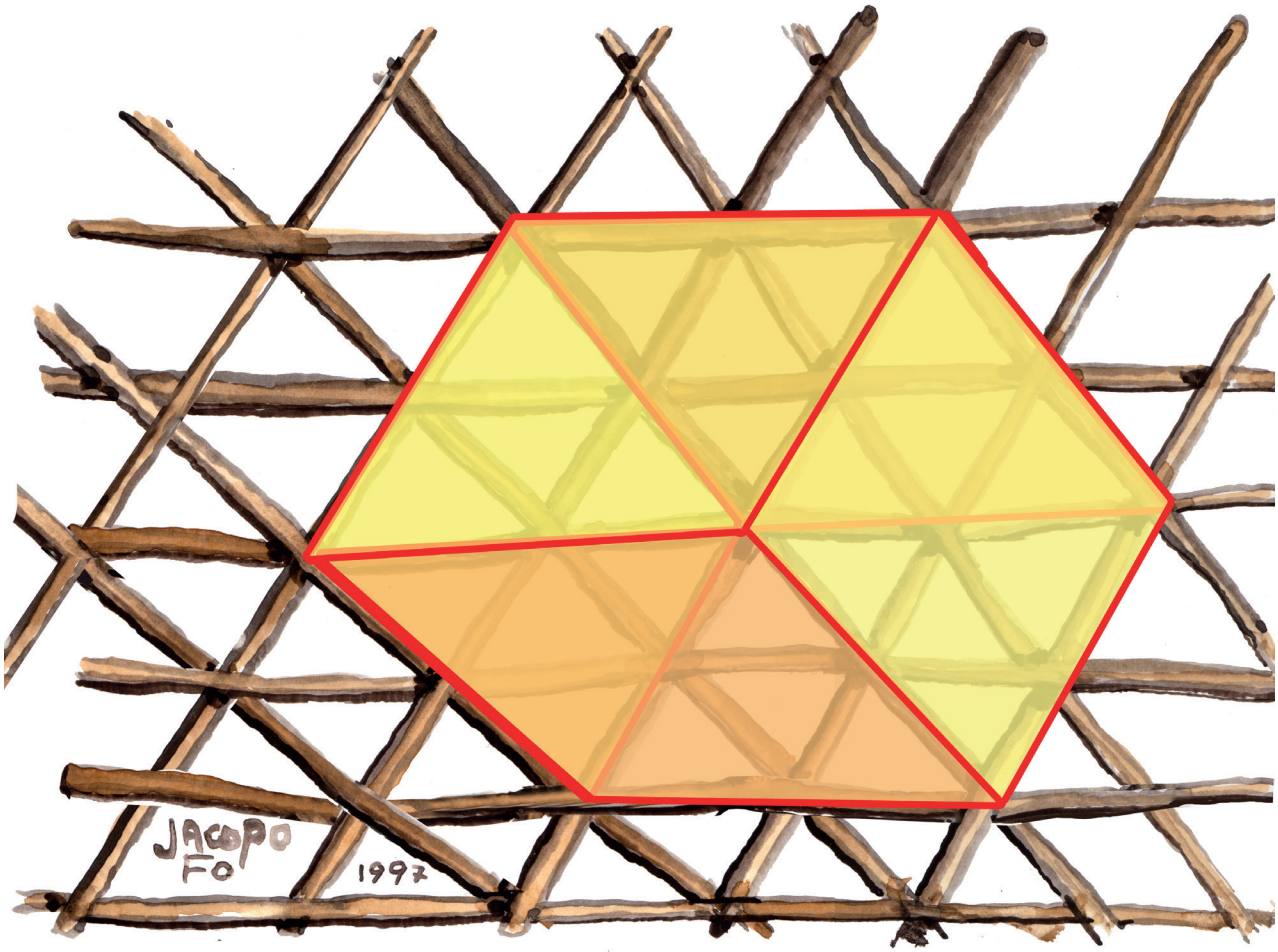
This is how the first calendar was invented, about 30 thousand years ago.



Palaeolithic Calendar

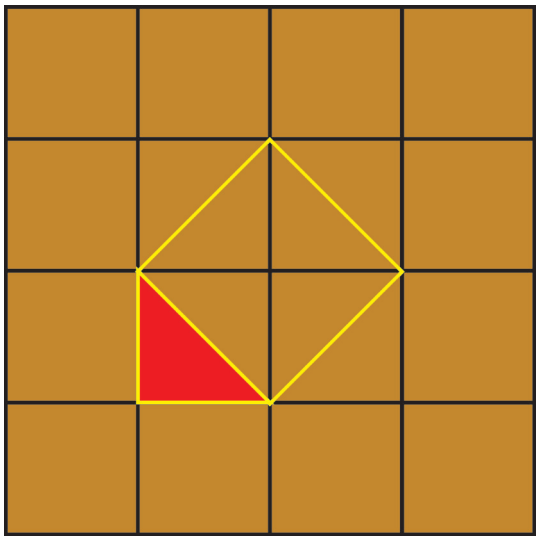
A long time after, someone, maybe weaving branches while building the wall of a hut, created a grid of triangles. This person then saw that more complex designs seemed to appear while gazing at the triangles. Hexagons, for example. And those hexagons could even seem to be three-dimensional objects: cubes.





This type of optical illusion made a great impression.

This is where the idea that the proof of world harmony, which was however invisible at first glance and had to be discovered, came from. Pythagoras's theorem, which demonstrates that in a right-angled triangle the squares of the catheti are as long as the square of the hypotenuse (which the Babylonians already knew), was considered as further proof of the existence of hidden harmony.

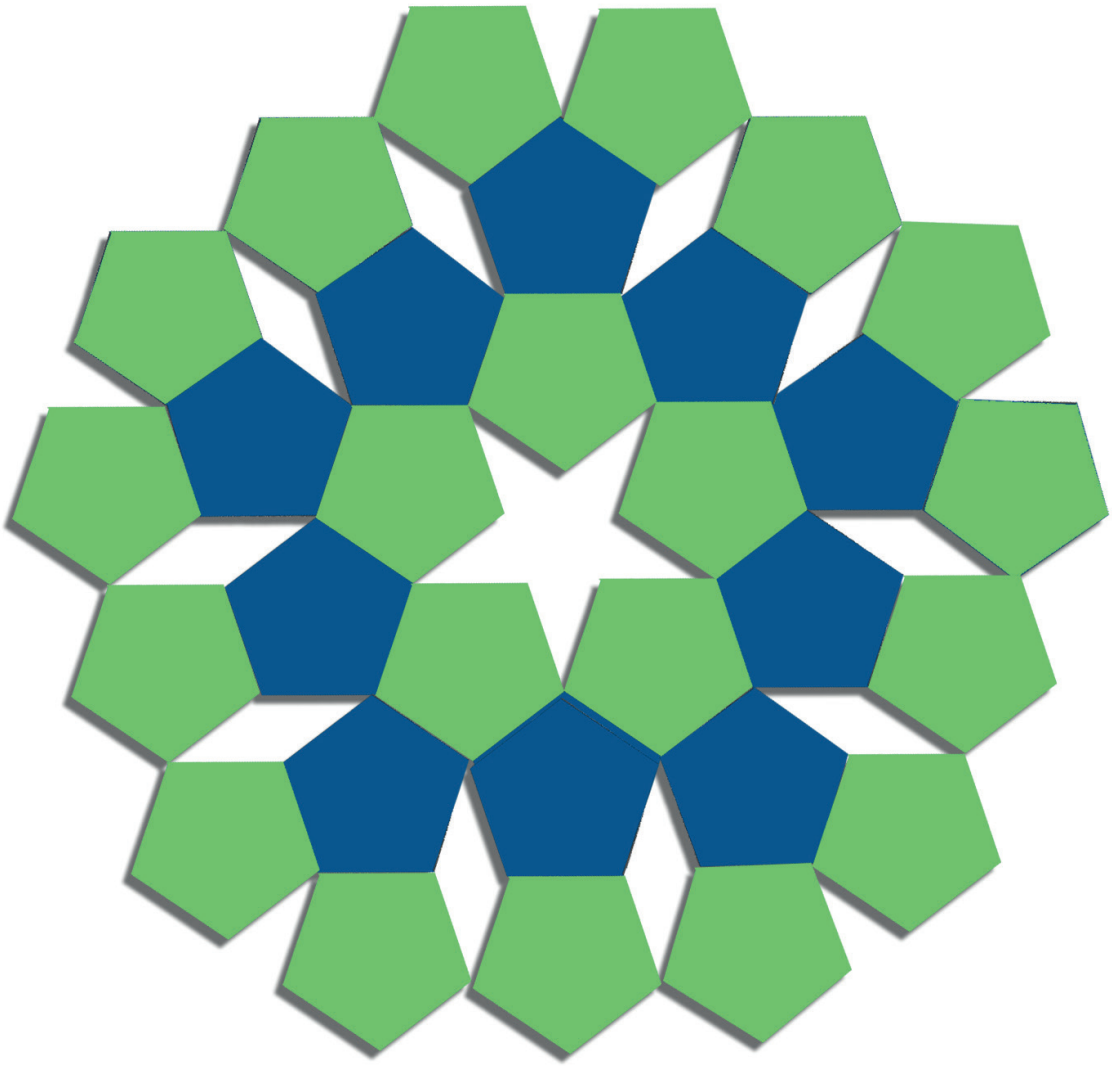


Centuries later someone started decorating temple walls with triangular and hexagonal grids.

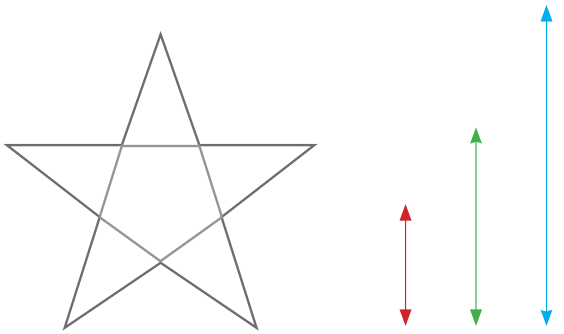
One day, someone else wanted to decorate a vase with a grid of hexagons, but realized that it was impossible because pentagon-shaped spaces appeared between them.

This seemed amazing and told of shapes that generate other shapes in the same manner as human beings generate children.

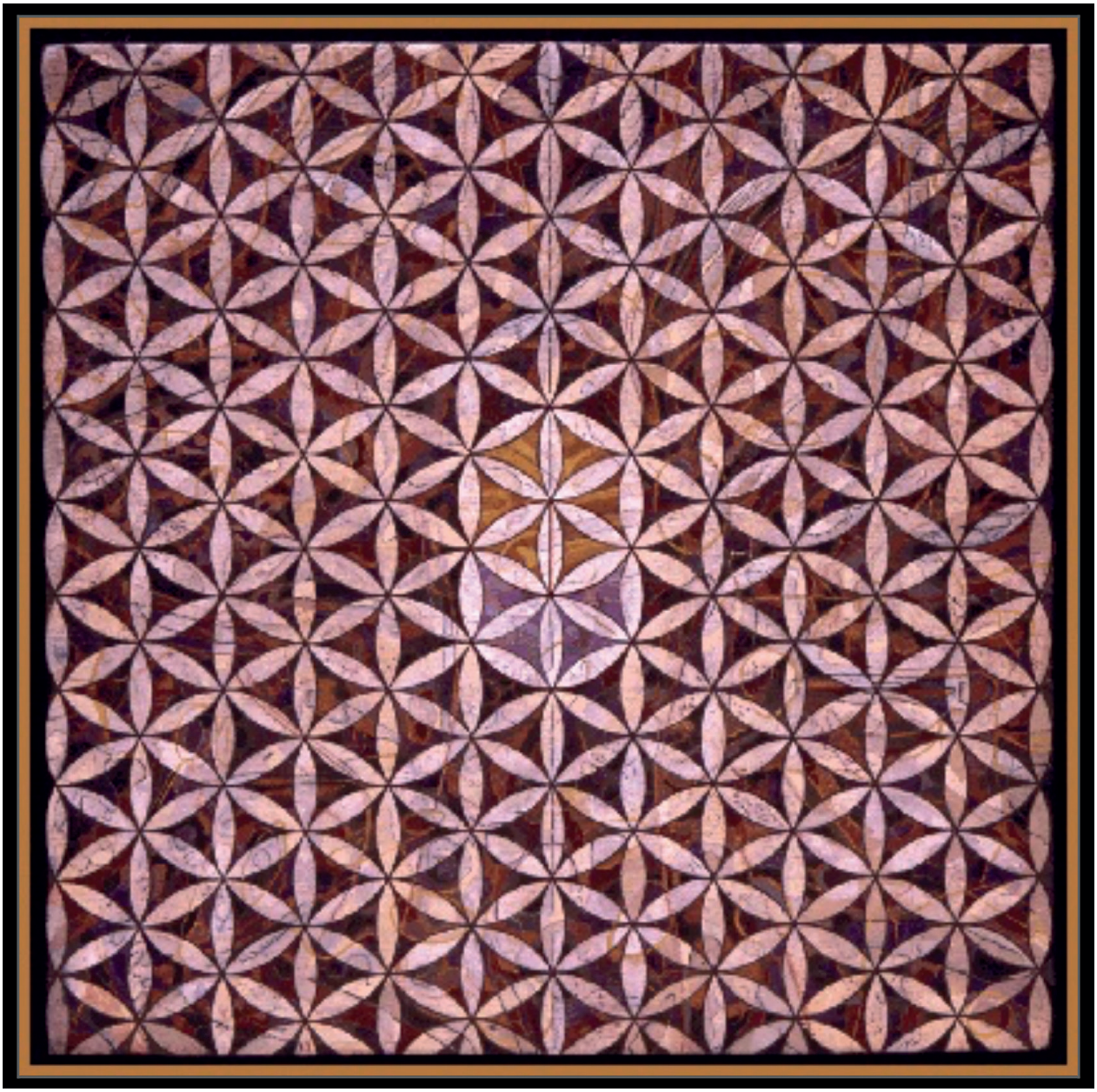
Another day, someone decided to decorate a wall with a grid of pentagons, but realized that even this was impossible; empty spaces remained, among them a 5-pointed star.



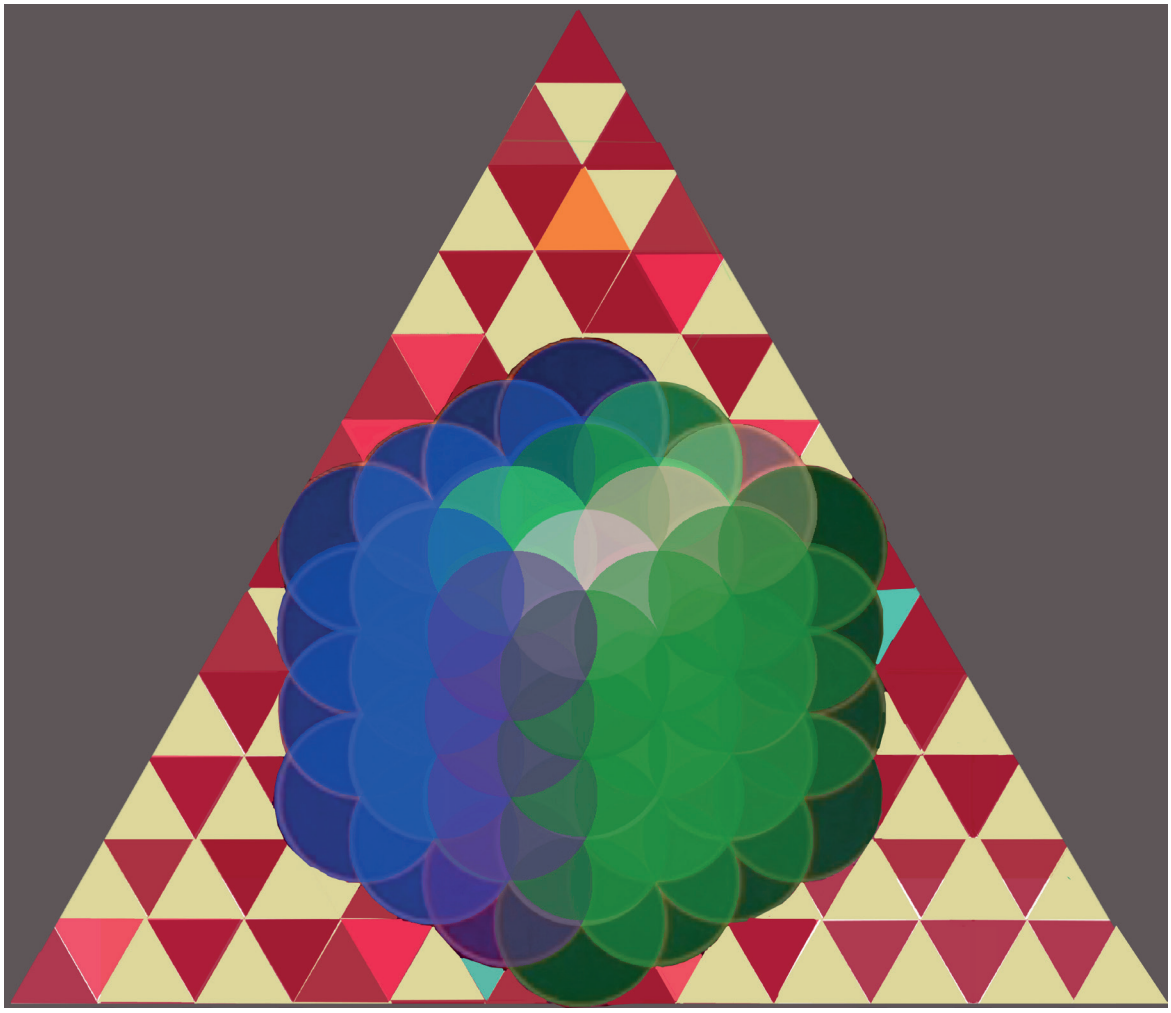
Then, maybe Pythagoras, maybe some much earlier than him, realized that the segments that form the 5-pointed star increased in size according to a fixed size ratio, and that this size ratio was found also in the phalanges of the fingers, in the distance between the leaves of some plants, in the layout of sunflower seeds...



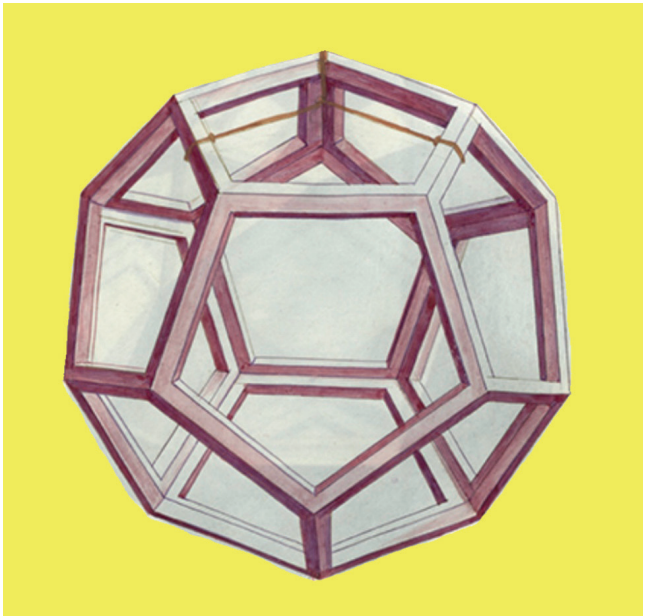
Subsequently, someone else etched a grid of triangles onto a clay tablet and traced circles on the triangles; the centres of the circles had the vertexes of each triangle at their centre. The image appeared as a field of flowers. Another strange visual effect.



And by squinting, it was possible to make out in that image even a series of cubes that grew in size, made up of 8, 16, 32, 64 spheres.

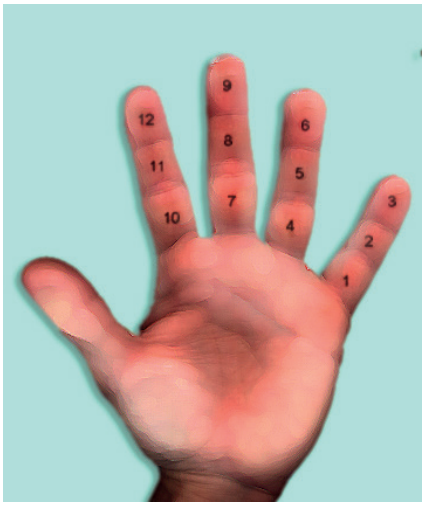


Others observed that a solid made up of 12 equal pentagons could be built.



A drawing by Leonardo da Vinci as drawn by Jacopo Fo.

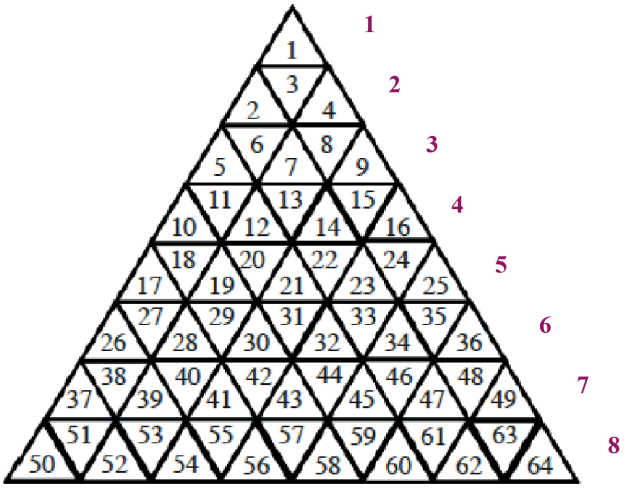
This was judged to be very interesting because in those times calculations were made using 12 as a base number, indeed people counted using the 12 phalanges of the 4 fingers of the hand after the thumb; the thumb was used to keep track while counting.



The number 12 was very important for the ancients. The 6-pointed star that appeared in the grid of triangles was made up of 12 triangles, a further confirmation of the fact that, in the relationships between numbers and geometric forms, some numbers appear continually; it was believed that those numbers were PERFECT and studies on them began. It was discovered that 1 + 2 + 3 = 6 and 1 x 2 x 3 = 6 and summing the numbers from 1 to 6 gave a result of 21. And for the reasons we will soon see, even 21 was considered a perfect number. All this was judged to be very interesting, also because there was not much to do in the evenings.

The Taoists then elaborated a model of the world centred on the opposition of two complementary polarities: Yin and Yang. These two symbols represented male and female, day and night, summer and winter, but the Taoists noted that there was another very important number: 3. The world had three dimensions, breadth, height and depth; three states, liquid, solid and gas; and when male and female united they generated a new creature. As a result they invented a symbol that united 2 and 3, namely a trigram made up of whole or broken lines: two symbols used 3 times. In this manner they obtained 8 possible combinations.

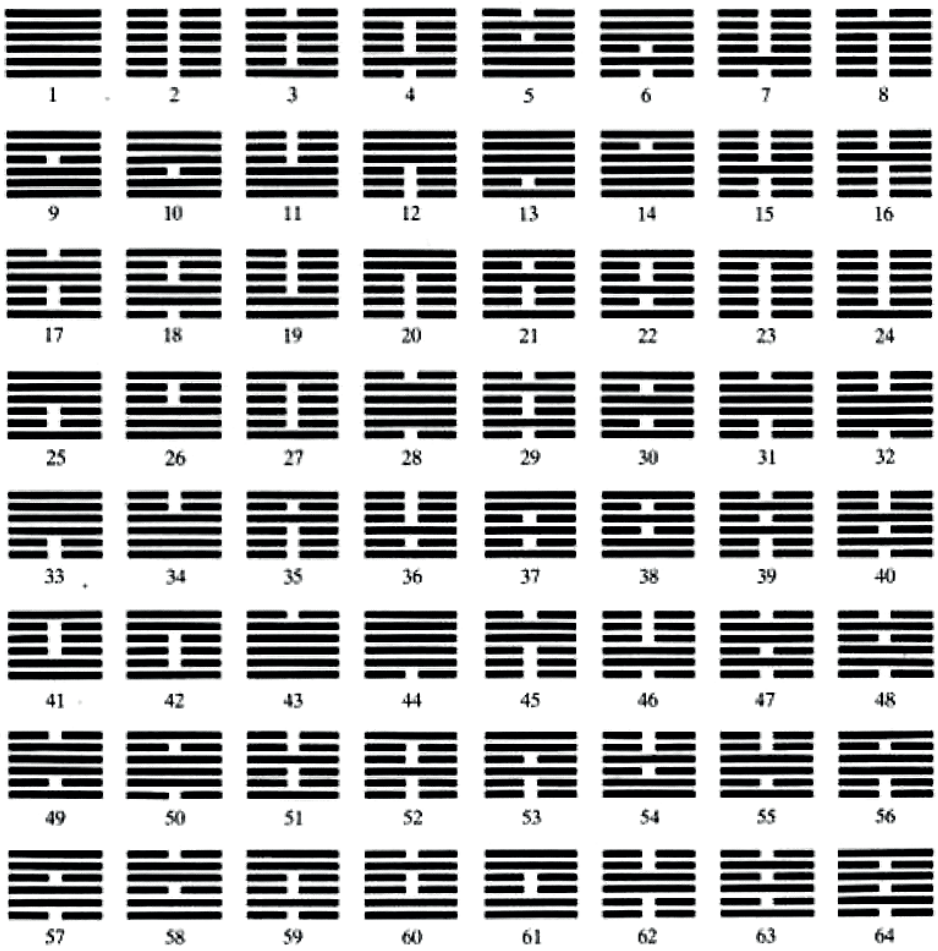
They then coupled 2 trigrams, forming 64 hexagrams. Even the number 64 turned up constantly. A Tetractys with 8 tiers is made up of 64 triangles, and the Flower of Life hides a cube with 64 spheres.



8-level grid containing 64 triangles.

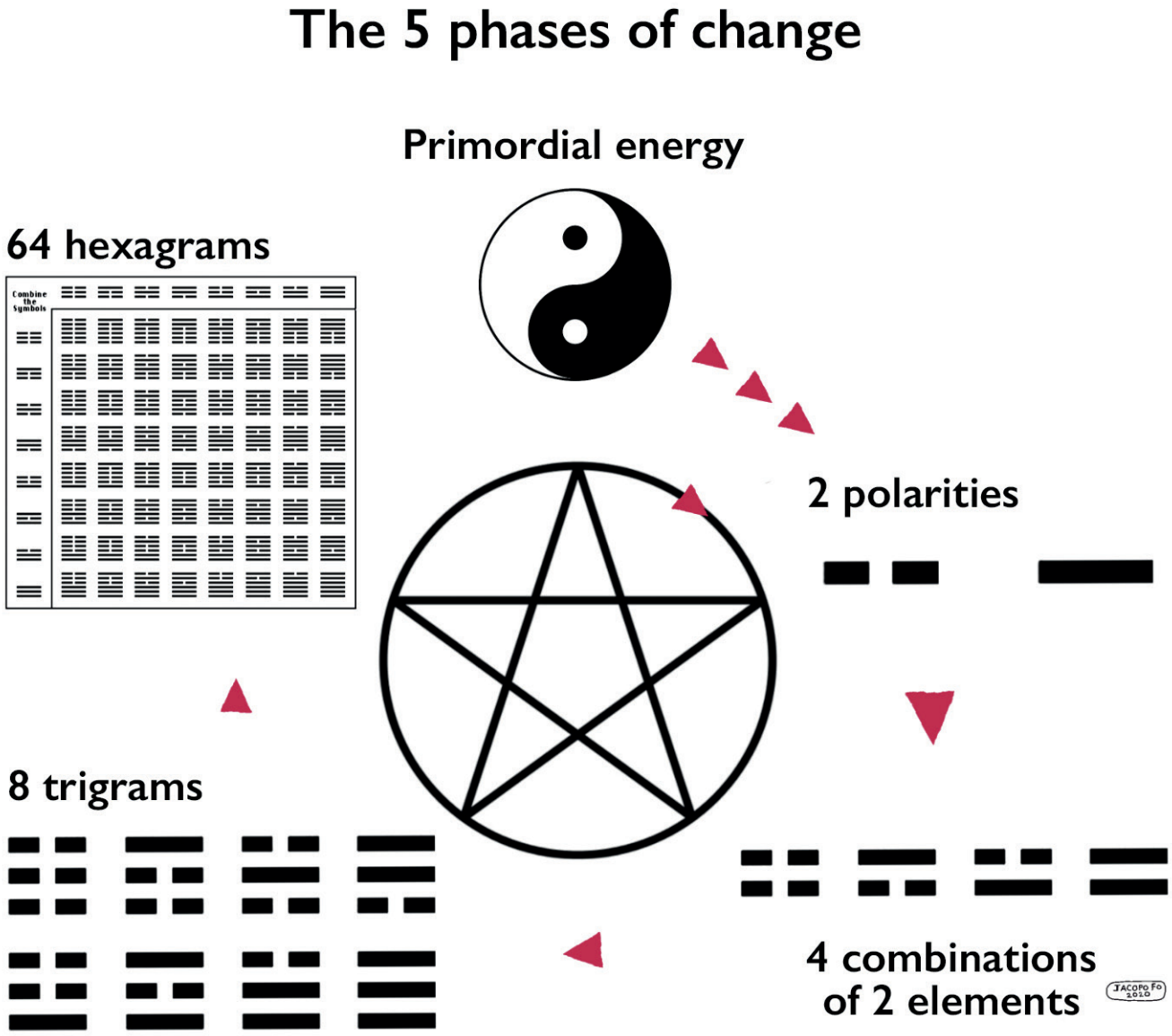
The Taoists subsequently developed this binary model by placing together two hexagrams (6+6=12).

This model was used, among other things, by doctors as a map of the state of health of the 6 innards and the 6 organs of the human body.



Re Wen sequence.

According to the ancient Taoists, there are 5 phases of change, which they represented with a 5-pointed star inscribed inside a pentagon.

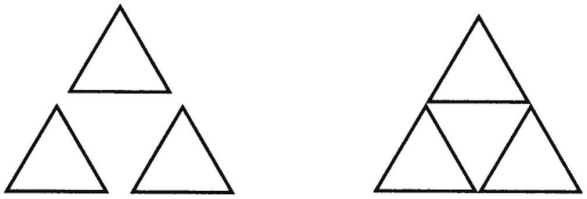


I believe that this account describes quite well the basic procedure that the ancient arithmeticians followed to lay the bases of their knowledge.

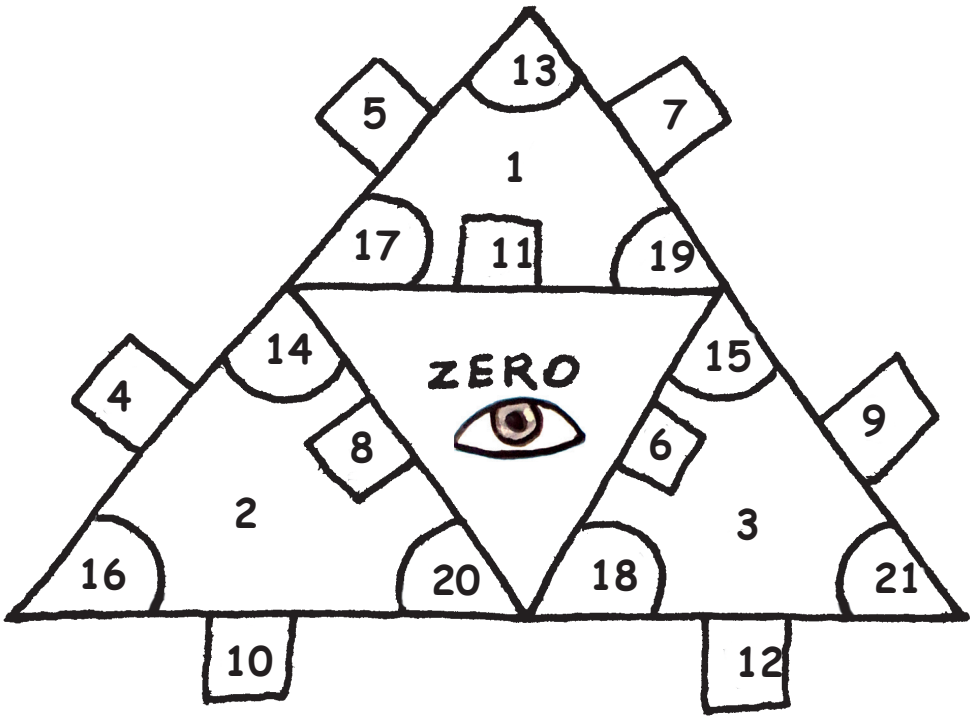
At a certain point, however, Pythagoras, the Israeli sages and the Taoists made a notable logical jump and discovered something similar to the idea of indefinability that is so dear to quantum physics.

The idea arose from the observation of what happens when three wooden triangles are placed together in such a way as to form a larger triangle. What can be seen is that the three wooden triangles form a fourth triangle that can be clearly seen even if it does not exist. A special relationship between 3 and 4 was therefore identified.

This observation seemed to fit well with the representation of the idea of divine trinity in the West, and in China it was used to illustrate the nature of the Tao that gives shape to things even if without material substance ("What is useful in the glass? The empty space it holds. What is useful in the door, the empty space that allows you to enter").

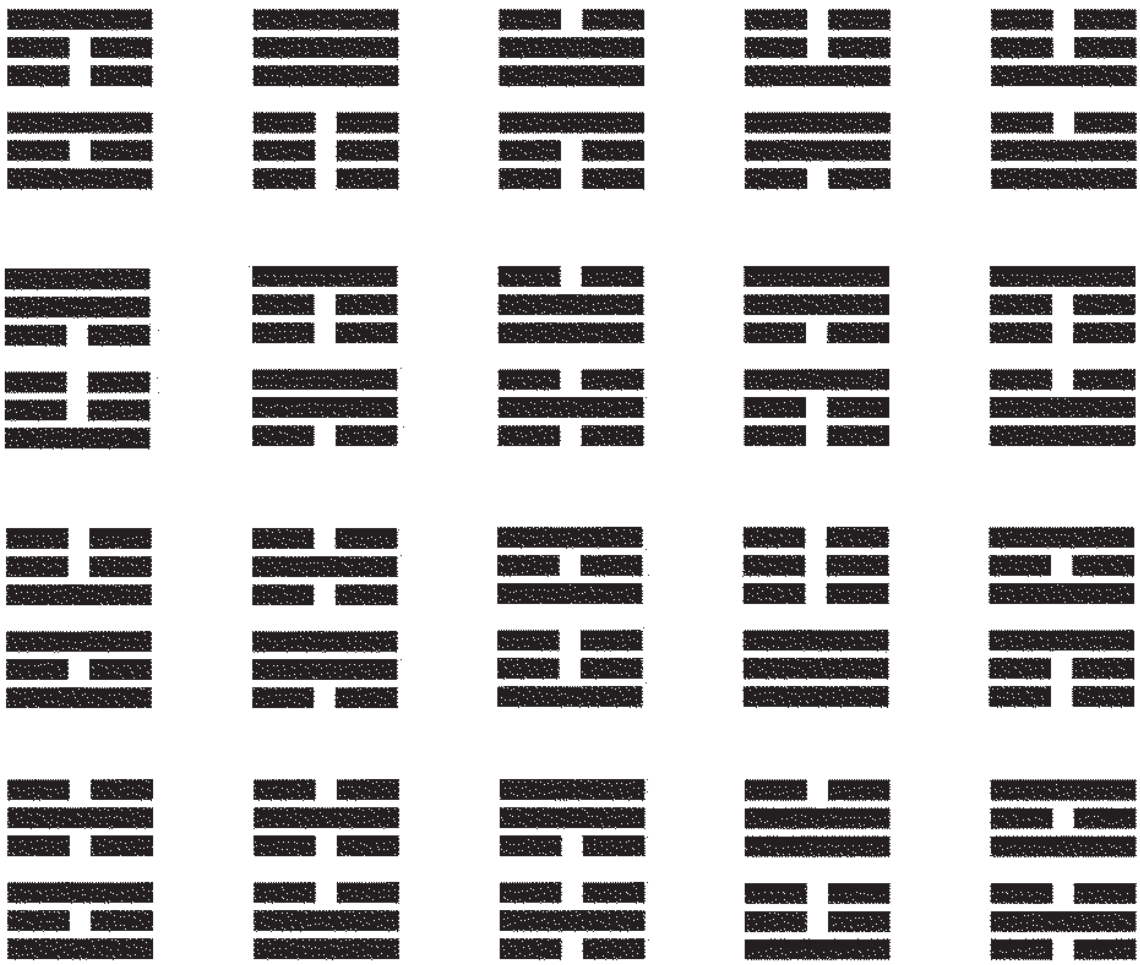


They also discovered that there is a similar relationship between the numbers 21 and 22. In nature, as in arithmetic, many entities can be divided into 21 groups plus one that exists only as an idea, a type of zero. Exactly like what happens in the Jewish alphabet. And they noted that those three triangles, the three which form a larger triangle when placed together, have 3 areas, 9 angles, 9 sides = 21 + the central area that does not truly exist and counts as zero = 22.



At this point the Chinese made another mental jump. Convinced that they had identified 64 as a number that regulated quantity, as we have seen, they represented it in binary terms using whole and broken lines. Modern mathematicians write binary numbers using zero and 1. They do not notice the graphic aspect of the numbers resulting from this numeric system. In the holistic vision of the Chinese, however, the graphic aspect became of fundamental significance. They considered the 64 hexagrams as a kind of ideogram.

They divided the 64 hexagrams into uniform groups. For example, the numbers that were expressed by the same amount of zero and 1 (broken and whole lines) were considered a family of stable numbers. This aspect became very clear when represented graphically.



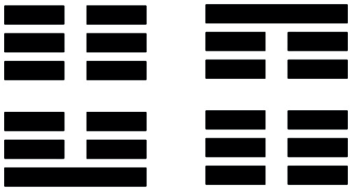
The 20 balanced hexagrams

The Chinese then identified other groups of numbers: for example those that were represented by the same hexagram but overturned or inverted (where the broken lines became whole and vice versa).

The fundamental observation they drew from this discovery is that there are 20 hexagrams made up of three whole and three broken lines. In the Chinese model they represent the smallest components of reality, they are perfectly balanced, so they do not unite with other particles.

44 hexagrams made up of a majority of whole or broken lines remain; they represent the infinitesimal particles characterized by unbalanced polarity and which can therefore unite with other unbalanced particles to form larger entities that are more or less balanced. The pairs that are not perfectly balanced can also unite with other couples, in search of better balance, forming in this manner all the substances that make up matter.

But if we observe the 44 unbalanced hexagrams we see that each hexagram has an overturned twin.



2 specularly inverted hexagrams.

As a result, the 64 hexagrams seem to be a series of 20 balanced hexagrams and 2 series of 22 unbalanced hexagrams, one series being the overturned version of the other, therefore they correspond perfectly. But in every series there is a hexagram with 6 equal lines and, as there is nothing that does not contain the two opposite poles in reality, these two hexagrams are only theoretical.

We therefore have 2 series of 21 real elements + a hexagram that is only theoretical = 22. A correspondence with the perfect numbers 21 and 22 therefore emerges, numbers which are at the base of the Jewish Kabbalah elaborated by the rabbis in Israel. Only two diagrams are apparently different.

At this point the only thing I can add is that the system of 64 hexagrams corresponds notably with the structure of DNA, and not only because the combinations between adenine, guanine, cytosine and thymine can form only 64 codons.

The fact that the 64 codons represent only 22 amino acids (selenocysteine and pyrrolysine, which we find coded in particular segments of mRNA, represented by the UGA codon and the UAG codon (stop signals), have been added to the 20 amino acids known in the past) also corresponds.

This happens because some codons are synonymous with each other, they indicate the same personal amino acid in the same way that some hexagrams are equal but overturned in polarity or reflected as in a mirror, or with the upper trigram overturned in relation to the one below.

The final question is: is the series of relationships between the numbers 1, 2, 3, 3/4, 5, 6, 8, 10, 12, 20, 21/22, 62/64 and the relationships between these numbers and triangles, hexagons, pentagons, 5- and 6-pointed stars, dodecahedron, Tetractys, the Flower of Life and hexagrams strong enough to create a coherent totality, the description of a fractal which lies at the origin of existence and spells out its development?

Evidently, for today's mathematicians, this relationship does not exist, or does exist but is just casual, or is not important, it does not reach the conceptual solidity of an algorithm or a theorem.

It seems to me that it contains a fascinating story and that it suggests a different way of reasoning and observing some quirks of reality.

I hope to have aroused your curiosity. I have written a small book that looks further into the questions mentioned here: you can download it in Italian free of charge.

Available on

http://www.jacopofo.com/sites/default/files/geometria-aritmetica-italiano.pdf

and in English

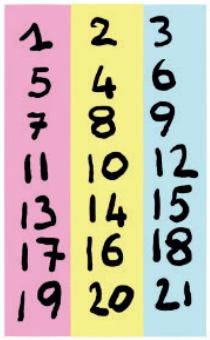
http://www.jacopofo.com/sites/default/files/geometria-aritmetica-inglese.pd

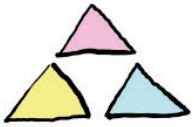
www.jacopofo.com.

Map that shows how Pythagoras' grid (Tetractys) generates hexagons, cubes, pentagons and 5-pointed stars and how triangular grids generate 64 combinations divided into 22 components in a similar way to the "words" generated by the 64 codons of DNA.

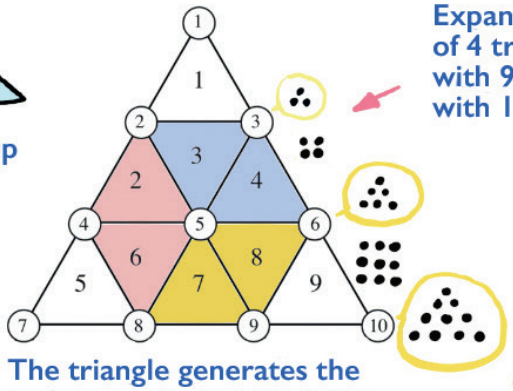
I can divide the numbers into: even and not divisible by 3; odd and not divisible by 3, divisible by 3

A	AREA	1	AREA	2	AREA	3
B	SIDE	5	SIDE	4	SIDE	6
	SIDE	7	SIDE	8	SIDE	9
	SIDE	11	SIDE	10	SIDE	12
C	CORNER	13	CORNER	14	CORNER	15
	CORNER	17	CORNER	16	CORNER	18
	CORNER	19	CORNER	20	CORNER	21
ZERO						

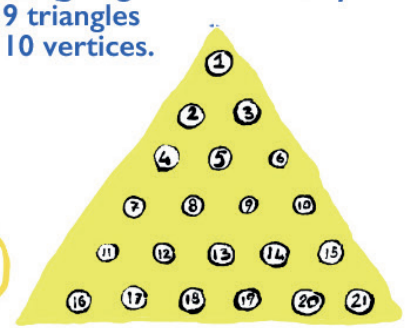




3 triangles alongside each other form 4 triangles with 9 sides, 9 corners, 3 areas + an internal area made up of the other 3 triangles with zero value because it does not exist in reality:
7x3=21+zero=22



Expanding the triangle made up of 4 triangles gives the Tetractys, with 9 triangles with 10 vertices.

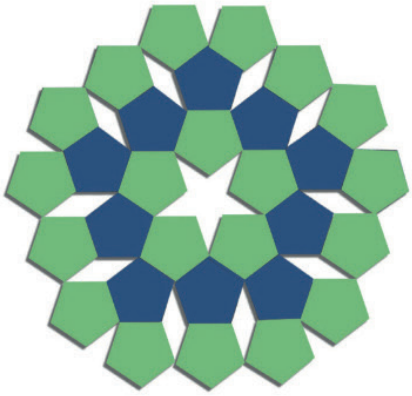


A grid of 5 levels has 21 points + the area where they are drawn, which has a value of zero o=22

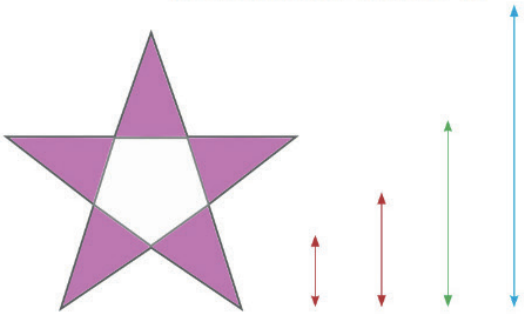
The Tetratcys contains a hexagon that can be seen as a concave or convex cube.

The triangle generates the hexagon 1+2+3+4+5+6=21+zero=22

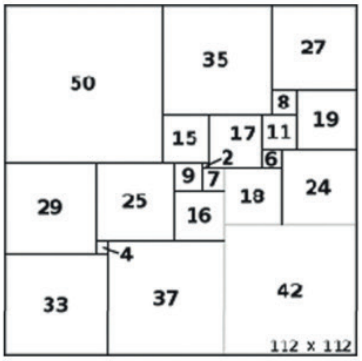
Hexagons drawn on a spherical surface generate pentagons.



Adjoining pentagons on a flat surface generate 5-pointed stars.

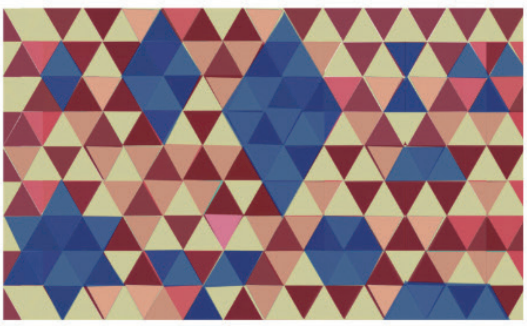


The sizes of the segments that make up a 5-pointed star correspond with the Fibonacci sequence.



1 square+21 inner squares of increasing areas in harmony with the Fibonacci sequence

The segments that make up hexagons, pentagons and 5-pointed stars are: 6+5+10=21 + the empty space they contain that has a zero value =22



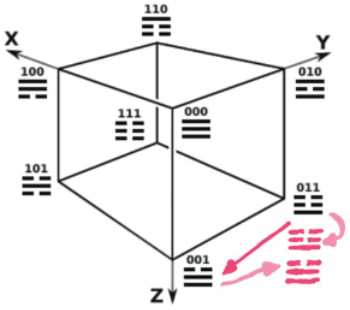
Triangles laid out in a grid generate different shapes in fractal form: 6-pointed stars, rhombi, hexagons, trapezoids, etc..
This led to the search, in this grid, for numeric and geometric properties similar to the Fibonacci sequence, which led to the discovery of the "function" that divides into 64 typologies and 22 components.

I can represent the relationship between 2 and 3 with a binary trigram that doubled generates 64 combinations

In the first part we say how the unit divides into 21+1 qualitative elements (tiles), like an alphabet with 22 letters, one of which a mute 'e'.
The succession of the units in numbering obeys the fractal matrix, which spells it out: 2<>3=trigram with 8 combinations, x 2= 64 combinations, x 64=4,096.

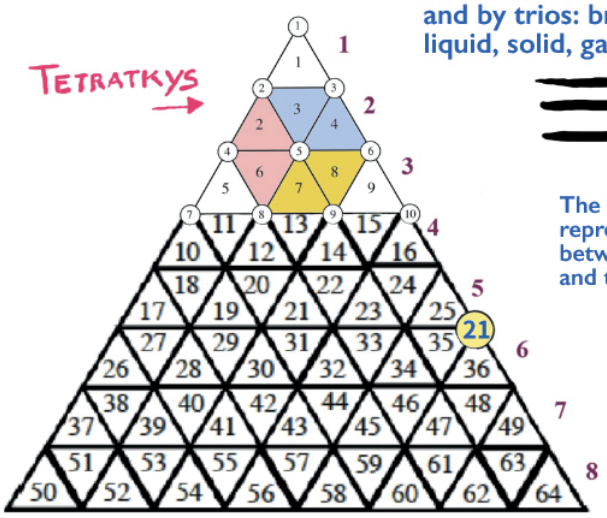
This procedure is similar to writing with ideograms which indicate the idea (phase) of each word instead of breaking down the sound into letters.
If we analyse the 64 hexagrams, dividing them on the basis of their qualities (balanced and not) or by the trigrams that make them up, we obtain a set of 22 elements.

The combination of two symbols taken 3 times produces 8 different trigrams. With these we can indicate the 8 vertices of a cube. 64 hexagrams are needed to indicate the roads between the 8 vertices.



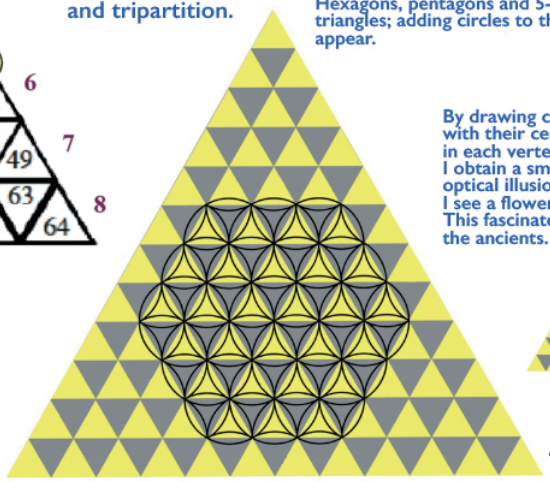
Reality is composed of pairs: male and female, day and night; and by trios: breadth/height/depth, liquid, solid, gas.

The trigram effectively represents the synthesis between duality and tripartition.

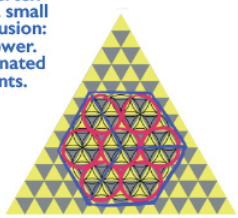


Grid, development of the Tetractys, with 8 levels, made up of 64 triangles.

Hexagons, pentagons and 5-pointed stars come from triangles; adding circles to the triangles makes flowers appear.



By drawing circles with their centre in each vertex I obtain a small optical illusion: I see a flower. This fascinated the ancients.



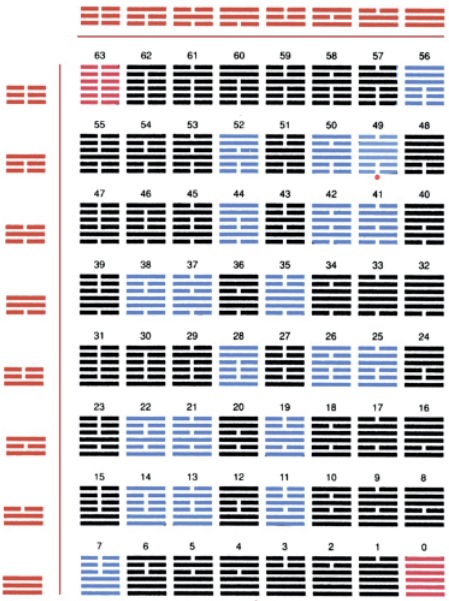
A smaller Flower of Life forms a cube with 8 spheres.

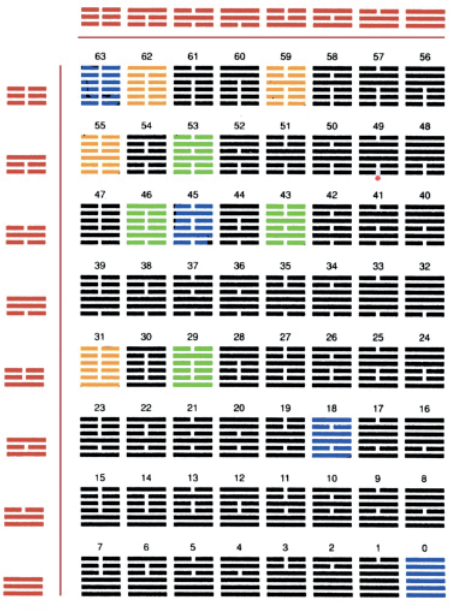
The Flower of Life

By uniting 2 trigrams I obtain 64, a binary numeration from zero to 63.

11

Combining 8 trigrams forms 64 hexagrams.





Division of the 64 hexagrams into 3 groups: balanced, not balanced and 2 virtual.

- 20 hexagrams with the same number of whole and broken lines.
- 42 unbalanced hexagrams (21+21=42)
- 2 only theoretical hexagrams with zero value 21+0=22

There are 12 groups of 4 hexagrams made up of the same trigrams but inverted or overturned; we can consider them synonymous with each other.
6 groups of two overturned hexagrams;
4 hexagrams that have non synonym.
12+6+4=22
This structure is similar to the 64 words" of DNA, which make up groups of synonyms and indicate only 21 amino acids.
(Biological tests often only list 20 amino acids present in the genetic code. However, selenocysteine and pyrrolysine were discovered recently, therefore there are 22).

Example of synonymous hexagrams because made up of the same 2 trigrams, either overturned or inverted

- Only theoretical hexagrams: nothing with only one polarity can exist.
- Equivalent hexagrams (with the same overturned or inverted trigrams)
- Equivalent hexagrams