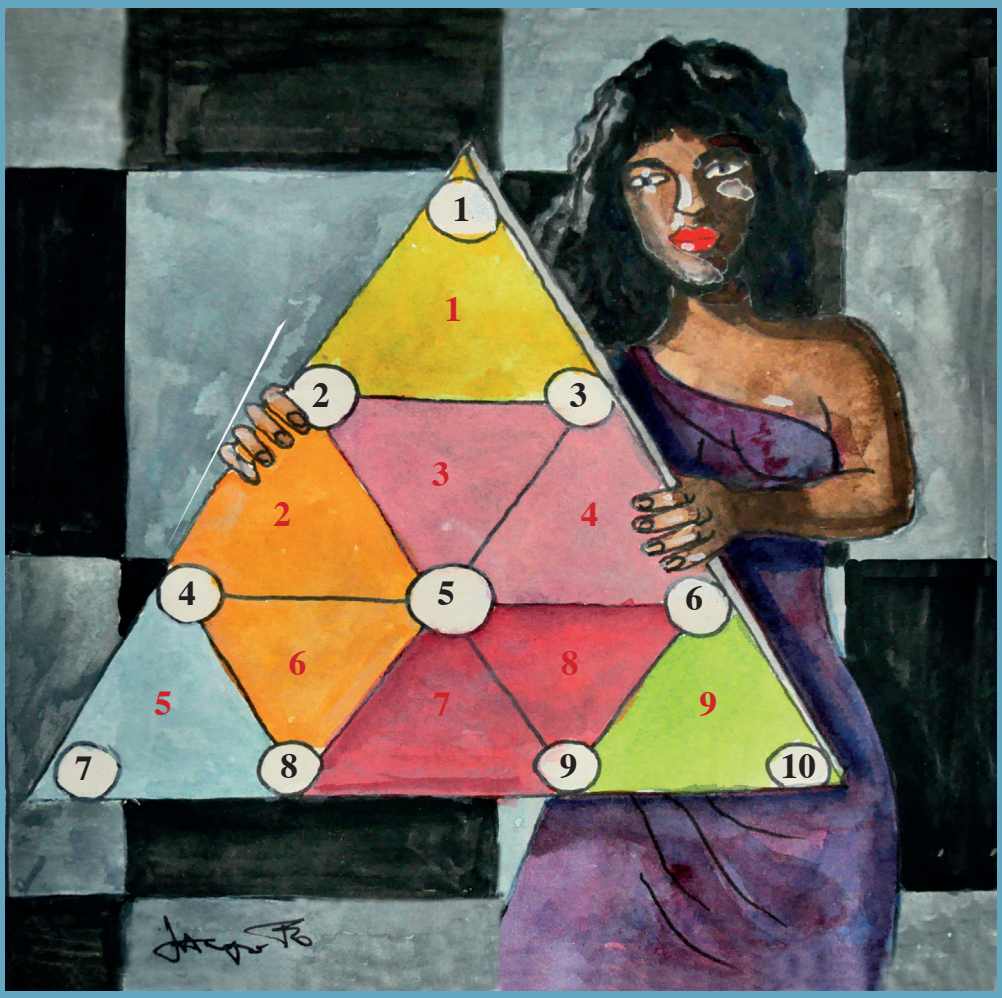
Jacopo Fo

Arithmetic geometry from Pythagoras to Israel to China

An investigation into the archaeology of thought



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CHAPTER 1

Pythagoras said that the Universe is made of numbers

The Greek maths teacher was not the first to confirm this. At least a thousand years before his birth, these ideas were known from the banks of the Nile to those of the Yellow River.

Pythagoras exposed this knowledge in a rational manner and for this was considered as the first mathematician in the modern sense of the word. Little is known about the model elaborated from his school, also because Pythagoreans had taken a vote of secrecy, believing that their discoveries could not be understood by everyone. And they were right, given that not even secrecy prevented serious persecutions. What could have been so terribly dangerous about their reasoning on triangles and pentagons?

Leonardo da Vinci and Isaac Newton put great energy into studying what Pythagoras called Arithmetic Geometry, but even they thought better of divulging their studies. Newton reached the point of burning a great number of notes to prevent them from becoming published, from destroying his fame as a great scientist.

As we will see, Pythagoras, Leonardo and Newton were afraid of spreading their discoveries for the same reason, namely that they clashed with the way of thinking of the period. Certainly, there is a great difference between Calabria in 500 BC and England in 1700, but the idea of the world of kings and priests was very similar in many manners.

Pythagoras understood that if he tried to explain his ideas to the king of Crotone or to the priests in the temple of Athena they would have beheaded him. Leonardo decided to keep quiet for fear of the inquisition, Newton to avoid the derision of the royal academy of science.

This motivated fear was not alleviated by the fact that their ideas derived from elementary realities that can be observed in the shape of triangles and hexagons, going beyond what normal eyes can see.

The research of the ancients was, in fact, extremely rigorous. It was a procedure that we could define as the precursor of the modern *scientific method*, because they considered only evident aspects, which even a child could see, without adding anything further than the ability to notice details that a hurried observer could miss but which, once identified, are solidly certain.

Few modern mathematicians are interested in exploring these ancient ideas because they smell of arcane magic, and even today mathematicians fear mixing with witch doctors, alchemists, magicians and charlatans, to avoid the contempt of the academic world.

As a result, today on the Web we can read thousands of pages dealing with the symbolic, literary, psychological or mystical meaning of numbers and geometric forms, and only a few pages dedicated to a simple description of the conceptual structure that gave rise to these symbols.

Yet there are many reasons to make understanding how our ancestors built their vision of the world worthwhile.

The forefather of Fibonacci and the hidden constant of numbers

In 1500 Fibonacci discovered that some numbers contain a growing proportion ratio that we find in nature *(Figure 1)*.

It is a constant involving the ratio between proportions, present in the curve of the spiral of the galaxies and in the spiral of a snail shell, that can scan the distance between the

branches of a tree or between the leaves on a branch, and you

can glimpse it in the layout of the seeds of a sunflower, in the shape of the viscera of a human being, in the curve that shapes waves, in the relationship between the length and thickness of each portion of the DNA chain. We can see Fibonacci's proportion ratio by observing the size of the phalanges of our fingers, or the width of our nose in relation to that of our mouth.

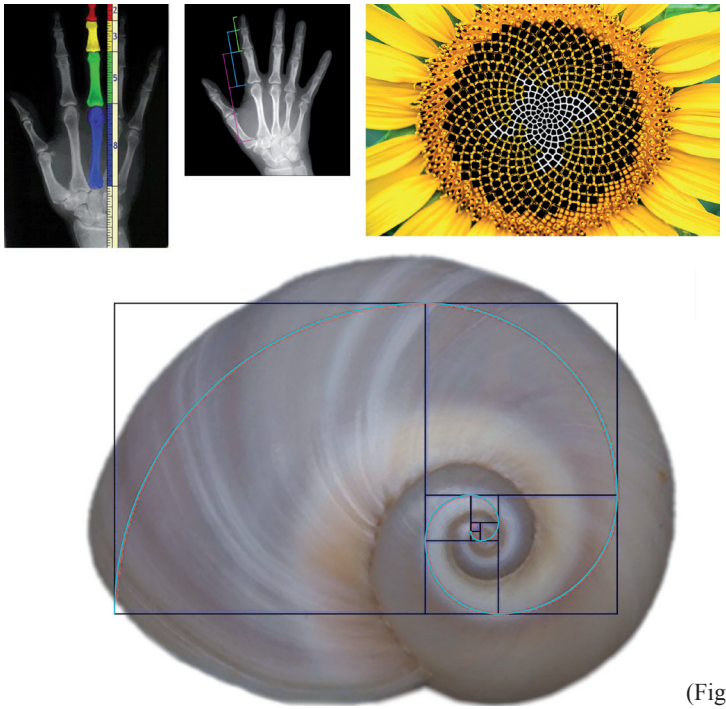
We can say that this proportion ratio is what nature selected to create many forms.

It is amazing that Fibonacci's sequence was known at least 2500 years before he described it using modern scientific language: many great ancient buildings were designed on the basis of these size ratios.

And I want to understand how our forefathers got there.

The discovery of these ratios between proportions is only one of the notable intuitions of our ancestors.

The idea that matter is made up of extremely small elementary



(Figure 1)

particles and that fractal forms are omnipresent spread among almost all the populations of the planet a long time before Greek philosophers started debating on the foundations of physics.

It's fascinating to discover why populations that were extremely distant from each other followed similar methods that led them to the same conclusions.

And how did these ideas mature without developed technology?

What follows is an investigation into the archaeology of thought. Using the surviving fragments of ancient knowledge, I will try to explain how they managed to build their model of the world.

CHAPTER 2

How did the passion for arithmetic geometry come about?

When starting this work, I first asked myself why our ancestors, even though their lifestyles, customs and cultures were completely different, had a passion for numbers and geometric forms, and how they were convinced that these things contained a trace for understanding the secrets of the universe.

Humans at the outset of civilization had to face numerous and serious practical problems, essential for their mere survival: tigers with curved fangs, shortages of food, parasites and illnesses of every type...what pushed them to enthusiastically reflect on numbers and triangles?

Why did numbers seem so important to them?

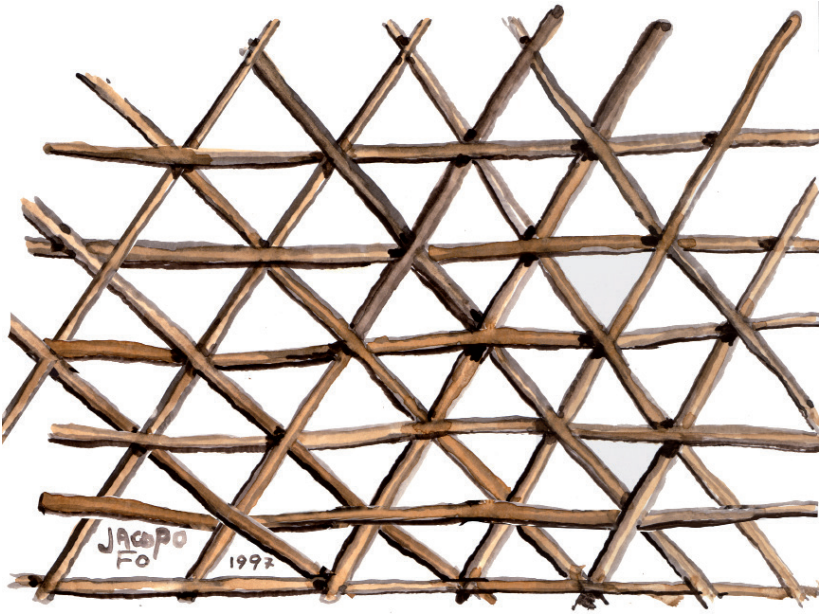
I discovered that the reason was actually quite banal: geometric forms seduced them because of a series of optical illusions that were casually present in front of them, and which they exchanged for magic, or maybe for messages sent by the spirits that governed the world.

Primitive humans probably encountered the simplest of these optical illusions when building the first huts.

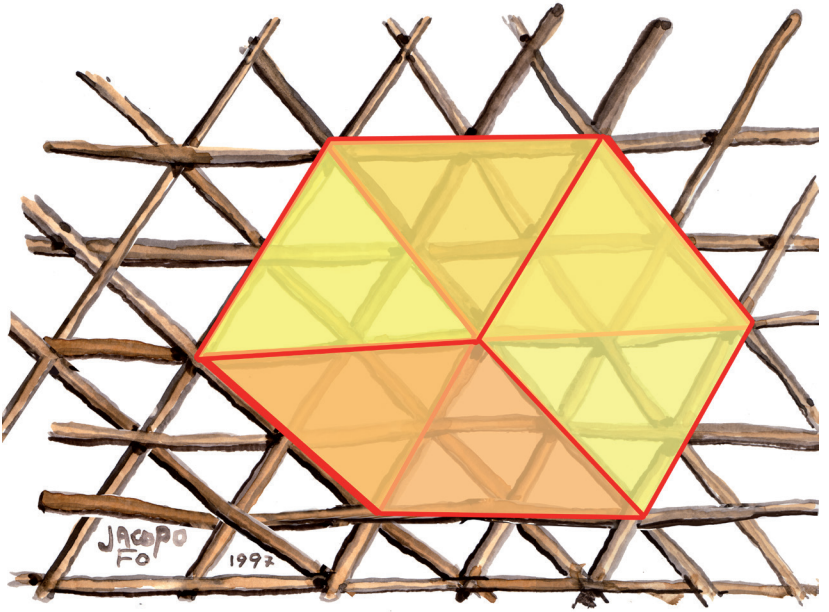
By crossing branches, in fact, it is easy to observe an unexpected three-dimensional figure: a cube. *(See Figures 2 and 3)*.

This optical illusion is insignificant for modern humans, but I suspect that in an era when next to nothing existed, this "special effect" had an impact similar to those that Hollywood's special effects have on us today.

Another surprise was added when they discovered other optical illusions as they explored the geometric forms generated by the triangular grids obtained from intersected branches.



(Figure 2)



(Figure 3)

It was this wonder, this illusion of experimenting something extraordinary, that led them to begin asking themselves questions, finding solutions and giving life, therefore, to the first form of abstract thought.

The mystery of Pythagoras

To understand how the ancients reasoned, we should start from Pythagorean ideas that contain the brilliant and genial summary of centuries of geometric observations.

There are two stories that help us understand Pythagorean thought.

Legend has it that one day Pythagoras, walking with his students through the streets of Crotone, heard the sound that some blacksmiths were making as they beat their hammers against anvils.

He entered the blacksmith's shop and began weighing the iron heads of the hammers and comparing their sound. He discovered, in this manner, that the sound made by a hammer is in harmony with that produced by a second hammer that weighs twice as much as the first.

In a single beat, he discovered that there was a relationship between the weight of a hammer and its sound, and that it was possible to build a harmonic scale thanks to a group of hammers with weights that were multiples of each other.

The idea that the Universe followed a law of shape and size regularity arose from this discovery.

It was a law of nature that had one particular point: it was not present everywhere constantly, but only dealt with some aspects. To better understand the power of the revolution of

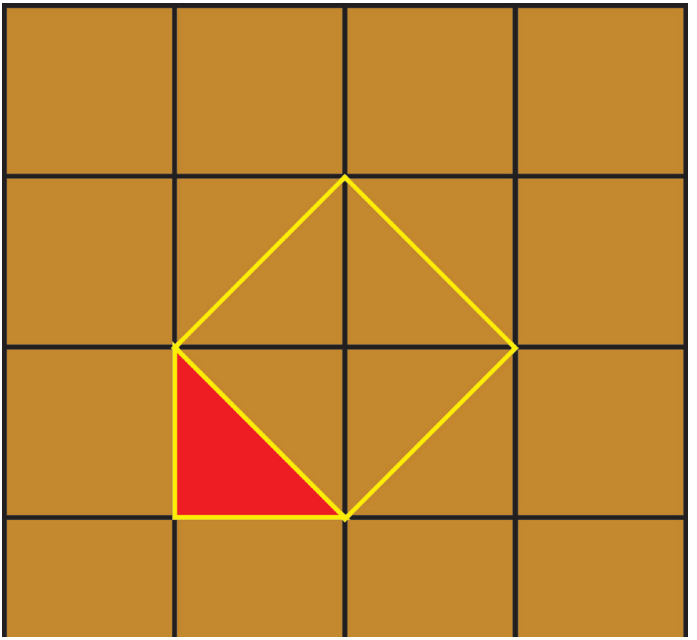
thought of Pythagoreans, we can recite another legend: Pythagoras was sitting in a room in the palace of the king of Crotone[1] waiting for an audience (normal bureaucracy...). While

1 - According to some, this episode occurred in the waiting room of the tyrant of Samos.

waiting, he started looking at the wall, which was decorated with refined square tiles. Maybe one was cracked from corner to corner, or maybe the sunlight produced strange effects, but Pythagoras noted a curious phenomenon: he clearly saw the demonstration of the theorem that was then given his name (and which made us all suffer at school).

For all right-angled triangles, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

In practice, he saw that half a tile was a right-angled triangle *(Figure 4)*.



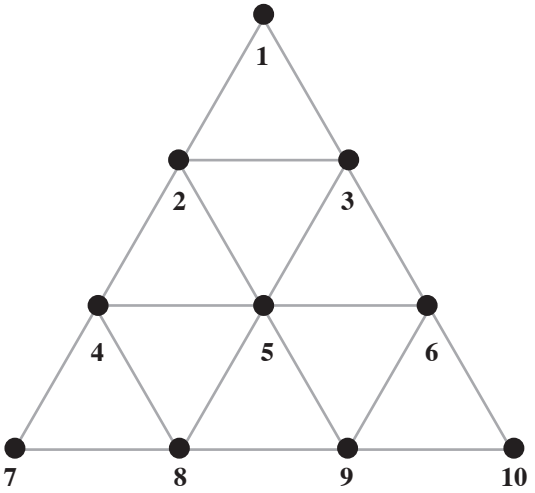
(Figure 4)

He then realised that the square which could be created along the long side of a right-angled triangle is formed of 4 half-tiles which, when considered together, form 2 full tiles. The exact size of the sum of the squares that I can create on the 2 short sides, namely 2 full tiles.

This discovery earned him eternal glory, but on the one hand he is attributed with a discovery that is not his, while on the other the most fascinating aspect of his observations were not recognised as being his.

Indeed, the theorem of Pythagoras was discovered centuries before his birth. We find it represented in a Babylonian tablet, shown here on the right. Pythagoras, though, understanding a new and extremely simple manner for demonstrating it, also discovered at the same time that a grid made up of triangles showed much more than Pythagoras' theorem in a simple and evident manner. After observing the tiles in the King of Crotone's palace and making his discovery, he rushed home and drew the numbers from 1 to 10,

Babylonian carving.



(Figure 5: The Tetraktys)

placing them in a triangle, and so creating a grid made up of 9 triangles. He called this triangle Tetraktys *(or tetrad, Figure 5)* and confirmed that it was the key for understanding the fundamental laws of the universe. After this, he established his school of arithmetic geometry.

Is Pythagoras' theorem shocking?

To understand Pythagoras' ideas, we first of all have to explain why the ancients thought it so important to discover parity between the squares of the short sides of a right-angled triangle and the square of the long side.

Why was it so fascinating to discover that beating two hammers, one double the weight of the other, against an anvil could result in harmonious tones?

At that time, there was no certain knowledge about how the world, the human body, natural phenomena worked. To humans from 2,500 years ago, the world appeared as an incomprehensible and chaotic set of occurrences, and a solid, curious mind could certainly not accept the puerile explanations which spoke of the sun being transported by a god on a chariot.

Pythagoras discovered that there was a rule, correspondence, harmony. It may have been hidden, but it certainly existed!

Apparently, there is no relationship of parity between the lengths of the sides of a right-angled triangle. But if I consider the squares created on these sides, then maybe I can find parity!

Those who are not mathematical will find it difficult to understand why this happens. It goes against practical logic but it is an undeniable fact: multiplying the lengths of the sides by themselves allows us to discover a parity that seems illogical at first sight. This evidence ensured that Pythagoras' theorem was considered as a kind of miracle of universal harmony by wise men in half the world.

And, as already said, Pythagoras went beyond demonstrating the theorem: he discovered that this demonstration emerged in a "natural" manner, simply by observing a grid of triangles. It seemed to Pythagoras that the shape of those triangles contained a principle that went beyond demonstration of the theorem.

Looking at that multitude of square tiles in the king's waiting room, Pythagoras saw a synthesis that could unify all the ancient geometric observations that had fascinated generations of thinkers and which could be found scattered here and there, untidily, on temple walls, in decorations on vases and other commonly-used objects.

After this flash of inspiration, Pythagoras created a theorem that considered sounds, forms and quantity as a phenomenon to be considered as a whole. Of his research, we know that he considered musical sounds, geometric shapes, the volumes of statues, the size of temples and natural phenomena to be permeated by a law of nature, of which the equality of the squares of a triangle and the proportion ratios that Fibonacci illustrated later on were only particular aspects.

For me to explain these ideas well, we have to leave Pythagoras for a while and take a leap back through millennia.

CHAPTER 3

How the optical illusions generated by triangles were discovered in the stone age

At this point we have to go back to talking about interwoven branches. There are some simple optical illusions that can be generated by casually drawing lines in the sand, or cutting the surface of a tablet of damp clay to decorate it, or interlacing long, straight branches to obtain the wall of a hut.

I am hypothesising when I say that the first geometric observations were a side effect of the technological revolution started when the art of weaving baskets and interlacing branches to build huts was discovered, techniques that led to a great leap in the daily quality of life of human beings.

The impression of this technological revolution can still be found today, in the cosmogony of the Dogon people in Africa (Mali). They believe that God created the universe by building a basket that contained everything. As a result, I am convinced that each thing is intimately modelled in the shape of a basket.

The Dogon people reached the conclusion that the structure of the weave was omnipresent. It is a very common way of thinking. All professional categories create an image of the world that is set according to their experience: plumbers see the world as a system composed of piping, flows and pressure. Carpenters see joints everywhere.

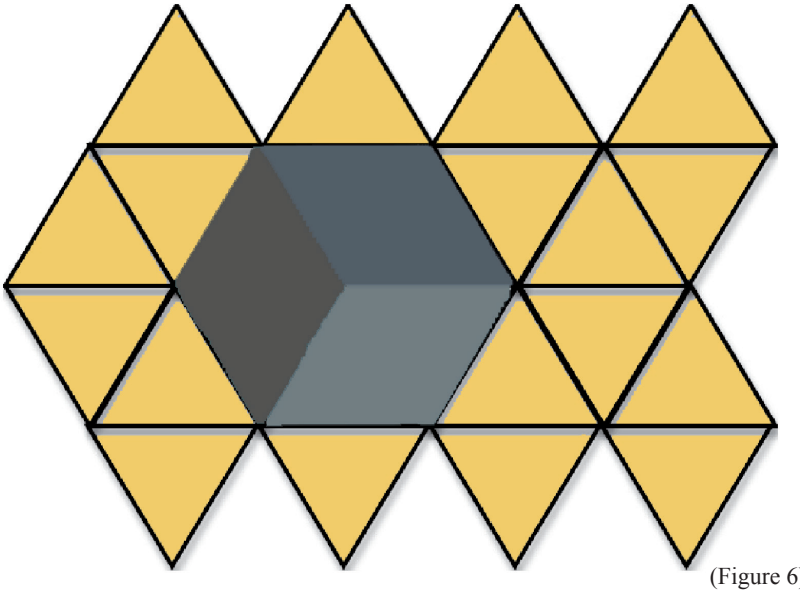
The ancient people therefore thought that everything in the world was woven.

This way of thinking would not have generated a complex model of the world if the form of the weave had not caused strange and amazing optical effects that gave those early researchers the sensation of coming into contact with something “supernatural”.

I triangoli stupefacenti

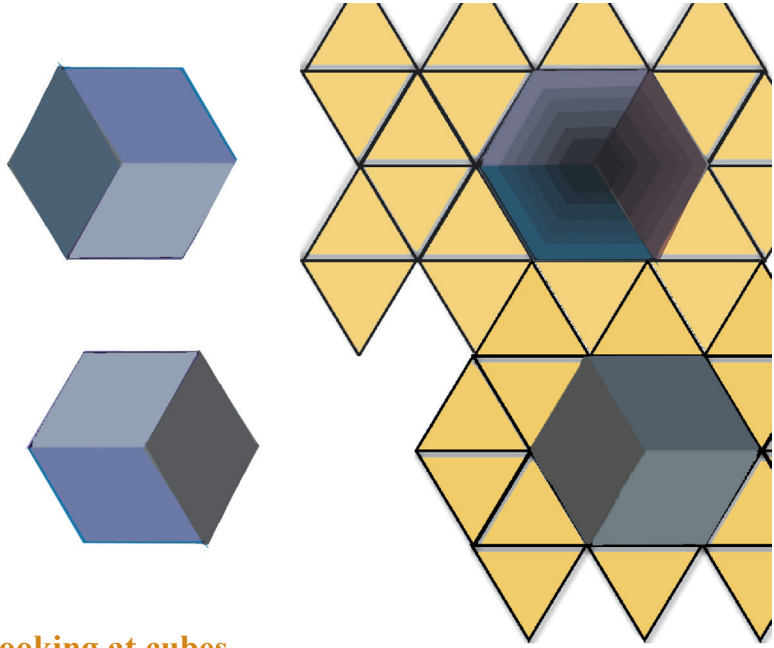
I imagine a human primitive building a wall made of reeds or branches, tying them together to create a grid of triangles. As already said, one day, maybe under the effect of some type of hallucinogen, he realises that in some points the wattle and daub seems to be protruding, creating a three-dimensional shape: a cube *(Figure 6)*.

His amazement increases when he realises that the cube did not appear in just one manner...according to how it was observed, it could be either raised or concave *(Figures 7 and 8)*.



(Figure 6)

Consider that there wasn't much enjoyment in those days, so in that cultural vacuum a hexagon made up of triangles that turns into **a cube with an ambiguous shape** seemed to be so wonderful and exciting that we find triangles in the decorations used by many ancient cultures.



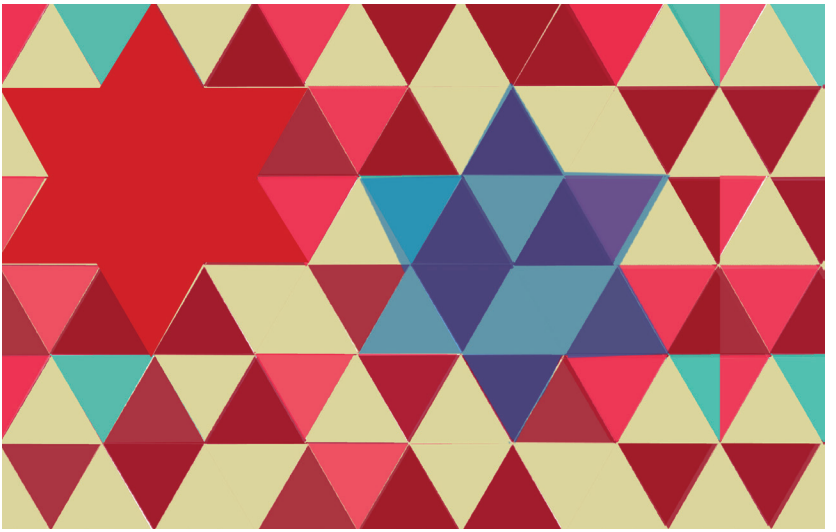
Looking at cubes that are lit in a different manner makes it easier to see the concave and convex forms.

(Figure 7)

(Figure 8)

Looking again at these sweeps of triangles close to each other, they saw another figure emerge, a figure that today is called the Star of David, which became a universally known symbol *(Figure 9)*.

This seemed even more amazing to them! And if you take a closer look there are also other figures: trapezoids, flowers, rhombuses...



(Figure 9)

Pentagons and five-pointed stars

The special effects that had the power to amaze are still not finished.

When the art of pottery was discovered, someone had the idea of decorating a vase with a series of hexagons/cubes lying next to each other. And he immediately realised that it could not be done!

The pattern that is so easy to produce on a flat surface cannot be reproduced on a spherical surface!

Drawing hexagons attached to each other on a spherical form

led to a completely ramshackle result: the hexagons no longer had 6 equal sides.

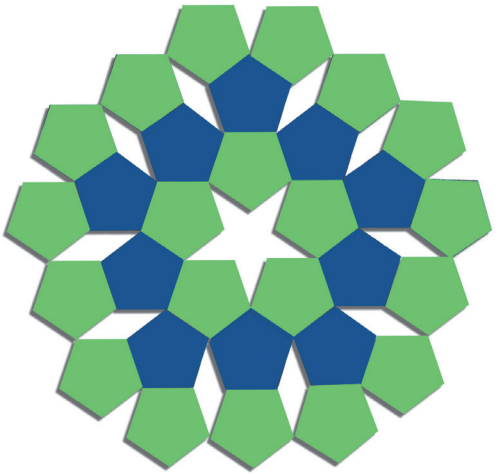
If the coherent form of the hexagons was to be maintained on the vase, a pentagon in the middle of the hexagons appeared automatically. It's the pattern you see on many footballs *(Figure 10)*. This is not because of an aesthetic stretch of the decorator's imagination, but because of the nature of the unquestionable laws of proportions and shapes. If a series of hexagons with 6 equal sides was required, a pentagon had to be positioned once every now and then to compensate the curve of the vase. Even this evidence struck the ancients greatly: the emergence of the pentagon was seen as a magnificent phenomenon.

(Figure 10)

This discovery led to a subsequent, extremely important discovery. Given that a series of hexagons drawn on a vase led to the appearance of a pentagon, they believed it to be a valuable geometric shape. They therefore decorated some vases with pentagons next to each other and discovered that it was a pleasant ornamental motif.

At a certain point, however, someone decided to decorate a wall with a series of pentagons placed next to each other. But when this person tried to place them alongside each other on a flat surface, he saw that it could not be done. No matter how they were placed, empty spaces of various shape appeared. One in particular seemed to be very interesting: the 5-pointed star *(Figure 11)*.

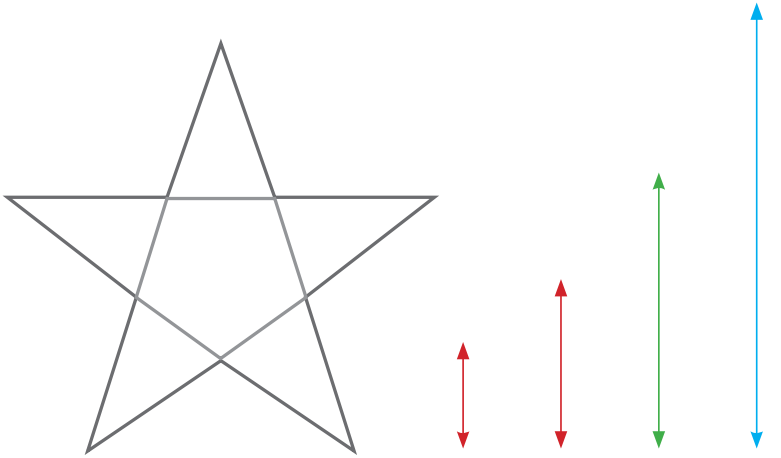
At this point, people had become skilled in reflecting on the re-



(Figure 11)

lationships between shapes. Triangles generate hexagons that are three-dimensional cubes, concave and convex at the same time, hexagons drawn on vases generate pentagons, pentagons drawn on flat surfaces generate 5-pointed stars.

The shape of 5-pointed stars was examined, and it was discovered that they contained a strange harmony... The ratio of the lengths of the segments that make it up is fixed. And it corresponds to the relationship between the phalanges of the fingers of a hand. It doesn't matter if the person's hands are large or small: the ratio between one phalanx and the previous one is the same as that

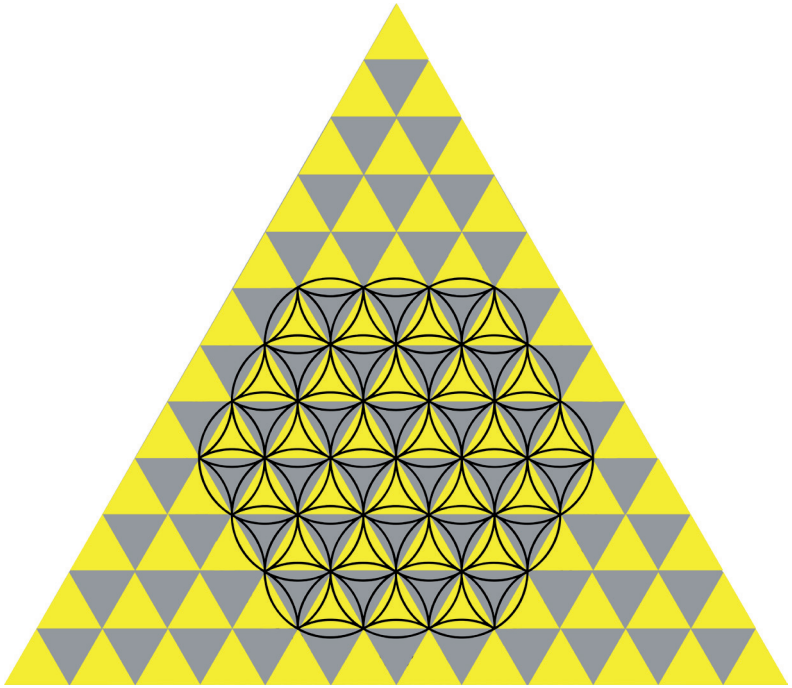


(Figure 12)

between the segments that form a 5-pointed star *(Figure 12)*. They looked around and discovered that this proportion ratio could be found in a large number of natural elements...they had discovered the divine proportions. From that day onwards, they tried to include those proportion ratios into what they built, and the statues they sculpted seemed more beautiful because they contained that harmonious ratio. They discovered that beauty and arithmetic and geometric coherence were connected.

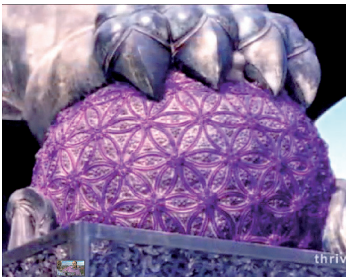
The flower that no-one drew

Then, one day, someone took a clay circle and, placing it over a grid of triangles, drew round it *(Figure 13)*. As if by magic, he obtained a flower with 6 petals, in spite of the fact that only circles

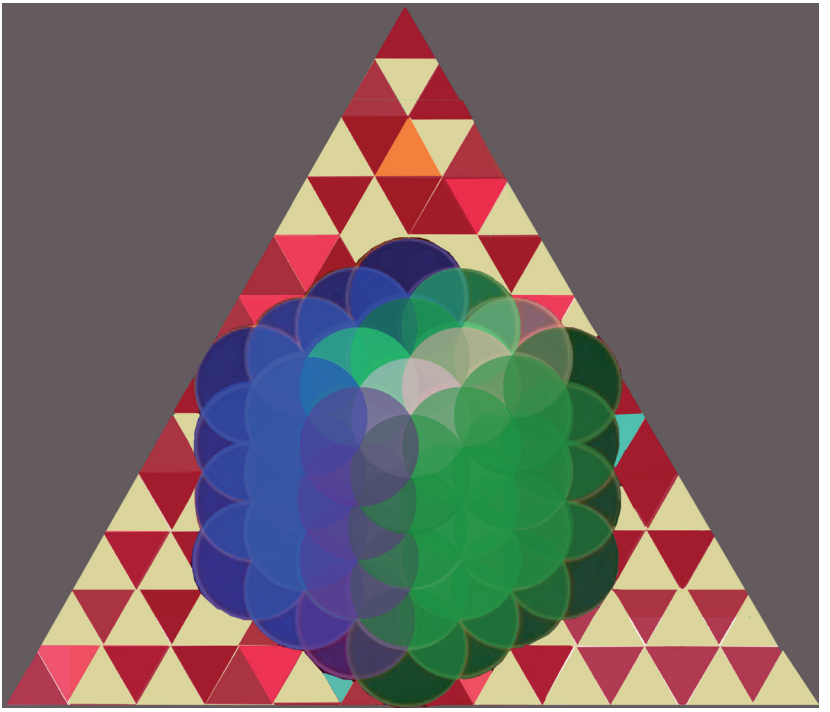


(Figure 13) The Flower of life is a very widespread symbol, from Egypt to China..

had been traced without any petal being drawn. It is another elementary optical illusion. Our mind is used to trying to recognise pre-set models. A sequence of circles and triangles has no usefulness.



The flower shape is advantageous to recognise, because our



(Figura 13)

ancestors ate flowers for millions of years...and some of them are delicious.

Our mind, therefore, identifies the pattern of the flower that seems to emerge spontaneously from the grid.

Even this flower amazed the primitives. Someone named it the

Flower of Life and gave it a great mystical importance, so we find it etched on stone in every corner of the world.

This emergence of the image of a flower from circles strengthened the idea that reality was based on fractal forms.

And another awesome fact was that by drawing many

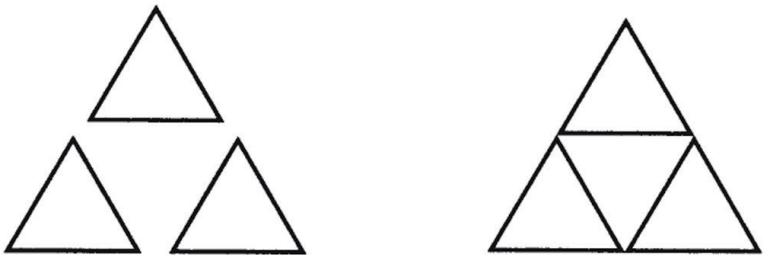
Flowers of Life on top of the grid of triangles, the flowers could unexpectedly no longer be seen, but a cube of 64 spheres appeared instead *(Figure 13)*.

The triangle that isn't there

At a certain point someone, reflecting on these geometric forms, noted another curious aspect: to draw a triangle divided into 4 triangles, you can draw the 3 marginal triangles without having to draw the fourth, because its shape is defined by the other three.

It doesn't seem to be very important, but it was extremely salient because it perfectly represented the idea that something immaterial existed in the centre of matter. This "centre" contained the essence of things. Chinese Taoists called this immaterial entity Tao, or Dao. Tao is everywhere but has no consistency. And it is the part that gives sense to everything. A jug would be useless if it didn't contain the emptiness that water can fill, and a door would serve no purpose if we could not pass into the empty space it delimits.

In the east, the ancient wise men convinced themselves that this image well represented the divinity, one and three: God in



the beginning was pure energy, then he decided to transform himself, creating the universe and as such manifesting his own three-dimensionality.

This simple design, only 3 small triangles drawn in the sand, confirmed the idea that the concepts that could describe the nature of creation were naturally enclosed in geometric forms. Arithmetic and geometry contained information that went well beyond their mere practical use of doing sums and calculating the area of a house. They contained a map of creation.

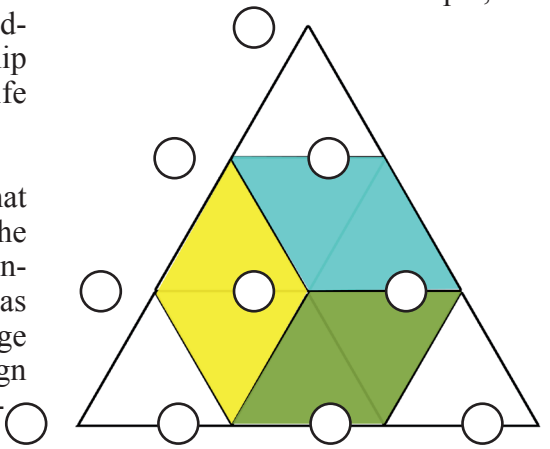
And all this series of coherent geometric forms that appeared, generated one from the other, created the conviction that there was a hidden harmony that revealed itself overwhelmingly thanks to its nature.

The lost knowledge of Pythagoras

Our knowledge of the way in which these ancient researchers attempted to understand the world is decidedly fragmented.

Millennia have passed, powerful but stupid men dedicated themselves to following men of wisdom, from Egypt to China immense libraries were burned, and men of science often decided to avoid trouble by keeping their discoveries secret.

Only some of Pythagoras' ideas have reached us. For example, we do not know if he meditated on the relationship between the Flower of Life and the laws of nature.



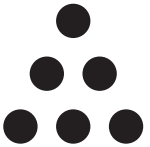
We are sure, instead, that he chose the Tetraktys, the grid of triangles that contains the hexagon/cube, as a symbol of his knowledge and the pentagon as a sign of recognition of the students of his school.

Many geometric and arithmetic observations have been passed down from Pythagoras' school, but little has reached us about the idea that they had of the world "made of numbers".

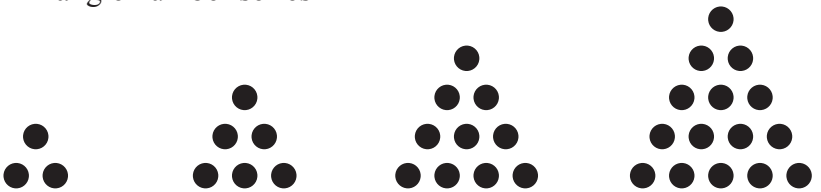
One of Pythagoras' ideas contemplated by modern mathematics is that of figured numbers. Someone probably did sums using pebbles placed in a row and realised that some numbers could form regular geometric shapes.

For example, with 9 pebbles you can form a square:

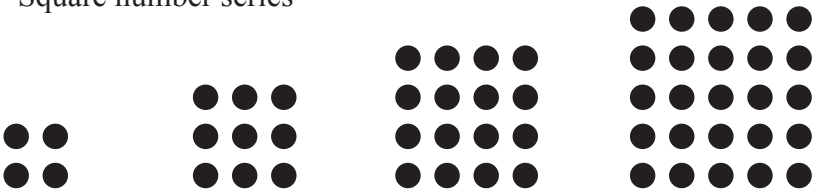
With 6 pebbles a triangle:



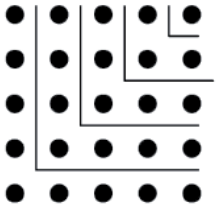
Triangle number series



Square number series



Modern, usual representation of squared numbers.



From this initial observation arose the idea of figurate, rectangular, pentagonal and even three-dimensional numbers, numbers that are cubed, pyramid, hexahedral, etc. Various properties of these numbers, correspondences, numbers that are sums and products of others, were observed. Today, figurate numbers are widely used to introduce children to mathematics, because they are fun.

I have not, though, found texts that ask what, in my opinion, is an essential question: why did Pythagoras find triangle and square numbers so interesting? Why did he consider them to be the key for understanding arithmetic geometry?

There must, also here, be an astonishing aspect in these numbers.

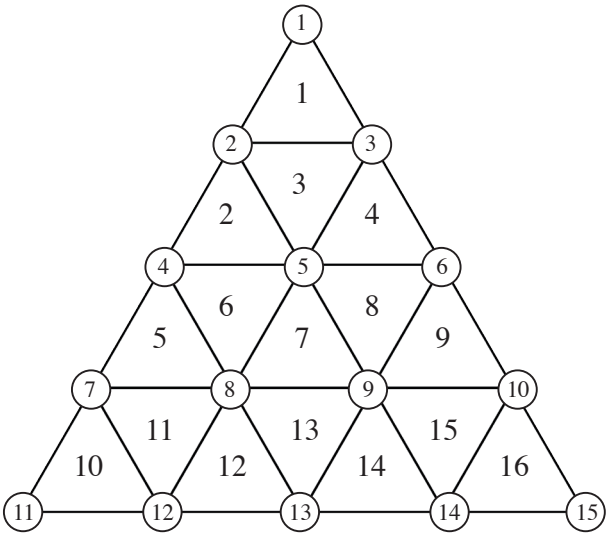
As we have seen, Pythagoras' research was based on those recurring and measurable aspects of reality that showed a subjacent harmony, which escapes every initial observation as in the case of Pythagoras' theorem.

From the relationship between square numbers and triangle numbers, an interrelation of the same type as that which emerges from Pythagoras' theorem should emerge, an equivalence which, at first sight, eludes but is subjacent...

Indeed, if we take the series of triangle numbers (1-3-6-15-21-28-36-45) and the series of square numbers (1-4-9-16-25-36- 49-64), the only connection that jumps to the eye is that 36 is both a square and a triangle number (continuing the list of square and

triangle numbers I discovered there are many numbers that are both square and triangle), but it isn't a lot for deciding to base arithmetic geometry on it.

However, if we take a triangle number expressed in points laid out in a triangular shape and we connect these points with segments, we find that each point is the coming together of these segments and that these segments mark out a series of triangles.



Taking the third triangle number, 6, I look at the drawing, I count the triangles it contains and discover that there are 4, which is the second square number.

The next triangle number, 15, produces 9 triangles and 9 is the next square number. And so forth. Writing triangle numbers in a geometric manner, I discover that they contain the square numbers. So, I discover that the square number is the sum of the number of triangle areas inherent to a triangle figurate number. At this point, the numerous complementarities between these

two categories of number become important, confirmation of a geometric harmony that penetrates reality. A fractal reality.

Triangle numbers	Square numbers
1	
3	1
6	4
10	9
15	16
21	25
28	36
36	49
45	64
55	81
66	100

Here is the diagram that contains the triangle and square numbers.

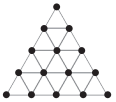
We are facing a discovery that is structurally similar to Pythagoras' theorem, a correspondence inherent to the geometric relationship (in this case between the number of subjacent intersections) in a number expressed with a series of dots, and the number of triangle areas inherent in this graphic representation, a relationship that cannot emerge in any other manner.

Finally, from observation of the Tetraktys, a triangle structured over 4 rows, derives the idea that 4 basic substances create everything that exists:

1- Unity, Fire
2- Duality, Air
3- Tripartition, Water
4- Materiality, Earth

The idea that everything was formed of just four elements was almost unanimously shared by all of Mediterranean culture.

Our knowledge of the Pythagorean model ends here.



Level 1. The upper point: the fundamental unit, completeness, totality, Fire.
Level 2. The two points: duality, complementary opposites, female and male, Air.
Level 3. The three points: the measure of space and time, the dynamics of life, creation, Water.
Level 4. The four points: materialism, the structural elements, Earth.

We know that the Tetraktys was the symbol and chart of the components and the relationships between the components of the universe, and represented the four basic components of creation. We know that Pythagoreans swore secrecy and faith on the Tetraktys and that the five-pointed star was their secret symbol of recognition. We know that the conceptual pillars of their model were arithmetic geometry and the harmonious relationship between the weight of hammers and the sounds they emitted, and as such the correlation between the harmony of the sizes of musical instruments and the sounds they produce.

But we know nothing about how this basic model was then developed to classify the nature of chemical substances, or the layout of the stars, giving rise to alchemical and astrological theories.

And if some urn full of parchments from 2500 years ago doesn't appear, we can only make suppositions.

Luckily for us, at the same time as Pythagoras' theoretical hammering, in the other part of the world, China, they were refining a theoretic system that is still being used today in traditional Chinese medicine.

We therefore have particularly precious evidence of the archaeology of thought. As we will see, this idea developed thanks to the discovery of another set of amazing aspects of reality that fascinated those populations.

And, *ça va sans dire*, observing this idea can help us understand how the Pythagorean model could be.

CHAPTER 4

The Taoist hexagrams

The theory of Chinese Taoists is based on two fundamental ideas: everything is made of extremely small, invisible, particles of energy and these particles follow a fractal development. These ideas were known even in India and in the Mediterranean basin, and the ancient wise men of China transformed them into a system for understanding reality, a type of translator.

Influenced by the discovery of pottery, even the ancient Chinese wise men convinced themselves that the universe was made from a type of delicate clay. Not just human beings: the whole universe.

Creatures, the sea, clouds, trees and swords were made from only one material, fashioned in a manner to take on different shapes. In essence, they reached a conviction that was extremely close to the reality of atoms which, joining together, form all types of material.

Probably by observing the hexagons, cubes, flowers, stars of David that emerged from a grid of triangles, they imagined that at the beginning of the universe there was a single form that had generated all the others: the triangle.

There was a third aspect that drew their interest: in the world, pairs are ever-present. Hot and cold, male and female, light and dark.

They therefore added a new idea to their model: there was only one substance that created the universe but it developed contrasting qualities as it evolved: at the beginning there was energy, Tao, that began showing itself with two polarities, Yin and Yang. They imagined that Tao had acquired yin or yang qualities

because it moved continually, such as water that evaporates and becomes clouds and the returns to the earth as rain. As such, when the Qi rose it was *yangYin*, when it lowered again it became *yinYang*.

Thanks to their different polarity, the particles of pure energy could unite and form more complex agglomerates, giving life to different types of matter. The changes depended on the varied percentages of yin and yang that they contained. Everything that existed was therefore classified as an entity made of yin or yang.

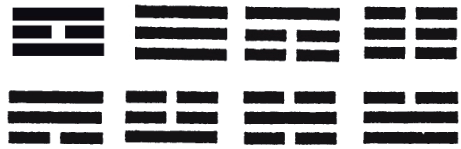
Male, heat, light, white, salty flavour, are yang.

Female, cold, dark, black, sweet are yin.

A system as simple as this one, based only on yin and yang particles, did not satisfy the ancient wise men. How could the diversity that we observe in the world originate from only 2 types of particle? They understood that each element has a ternary quality (height, width, depth; liquid, solid, gas). They thought, therefore, that Yin and Yang particles each had 3 qualities.

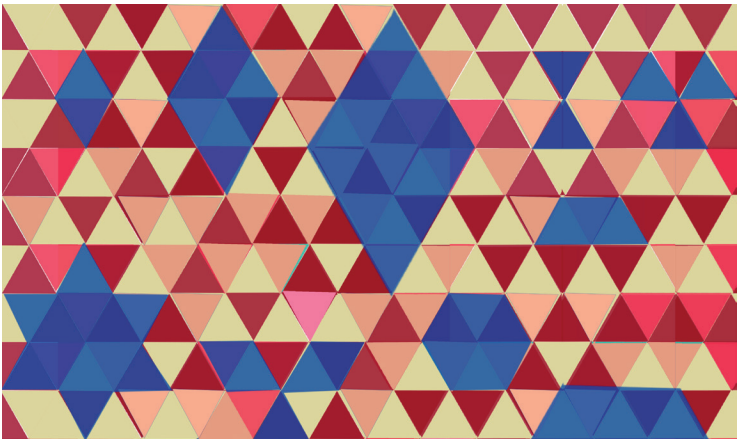
This model reached its maturity when it was expressed with a series of three lines of two types. In this manner, they obtained 8 particles: 4 dominant yin and 4 dominant yang.

This representation had an essential peculiarity. The position of each whole or broken line had a meaning because it referred to a particular aspect.



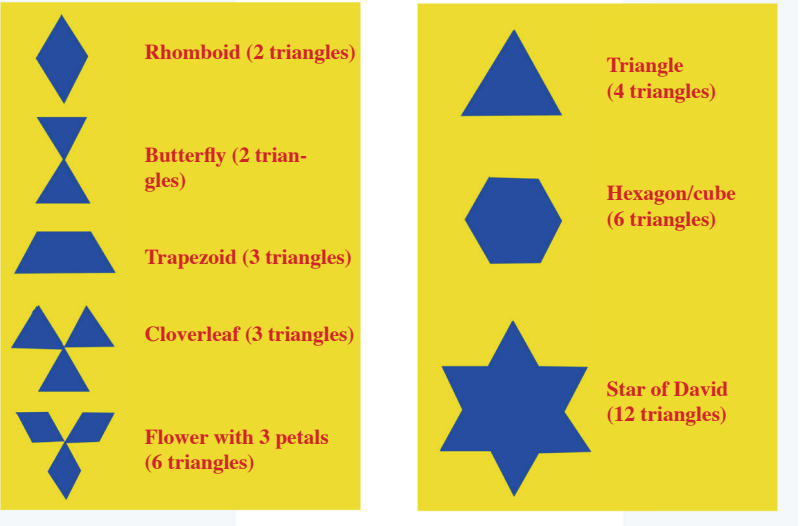
In other words, trigram  and trigram  , even though they both have 1 full line and 2 broken ones, have different meanings because the spatial difference of both lines is different. This difference depends on the fact that each line refers to a different quality (we will see this concept in a better manner in the next chapter).

This idea can be confirmed by the observation that 8 different forms could be seen in the grid of triangles *(Figure 14)*.

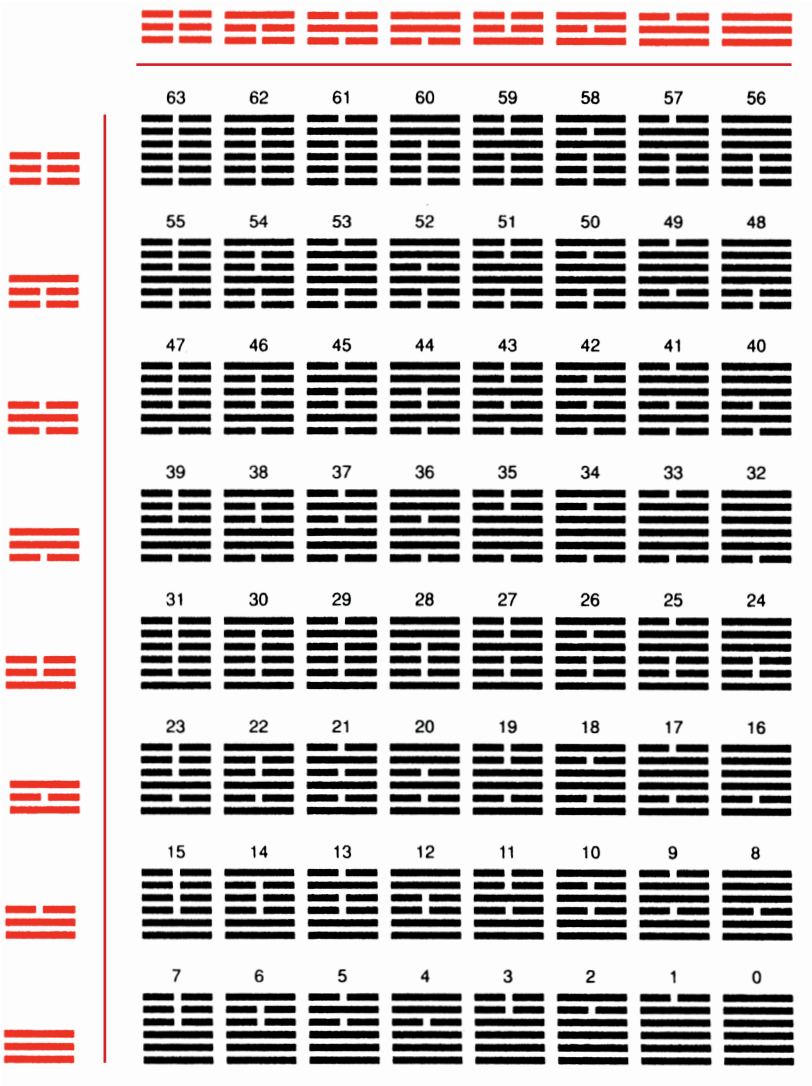


(Figure 14)

The 8 forms generated by the grid of triangles

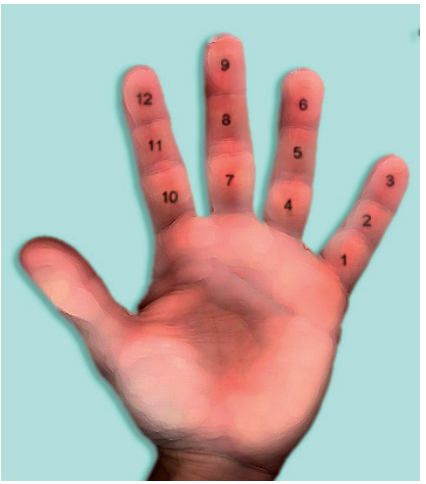


They therefore hypothesised that these particles would pair 2 by 2 once again, so obtaining 64 combinations composed of 6 stacked lines, called hexagrams.



Layout of the 64 hexagrams according to the model known as the *Binary or Fu Xi* sequence. This layout is based on the most logical manner of writing the hexagrams one after the other.
It is similar to our times tables. The 8 top trigrams are the same in every vertical column, the bottom trigrams are the same in every horizontal row.

Finally, they hypothesised that each chemical substance, plant or living being could be represented by a pair of hexagrams.



(Figure 15)

In this way, they could indicate each thing with 12 full or broken lines, and this conformed with the fact that the Chinese, just like the majority of populations, initially counted on a duodecimal basis, a system based on the habit of counting the thumb and the 12 phalanges of the other fingers *(Figure 15)*.

The system of the 64 hexagrams is fascinating from many points of view. Indeed, the 8 trigrams contain the discovery of binary numbering: by replacing the full or broken lines with ***0*** and ***1*** we obtain the binary numbers from 0 to 7. The 64 hexagrams correspond to the binary numbers from zero to 63.

The 64 hexagrams then became an oracular instrument and are today famous as the Book of Changes or **Yì Jīng** *(I King, I Cing or I Ching)*. Their initial purpose was not, however, to predict the future.

Taoists used these 64 hexagrams to catalogue the organs of the body, illnesses and cures; by observing how this cataloguing operates, we can understand their idea of arithmetic geometry as a fractal that creates the bones of reality and which, precisely for this reason, continually emerges in the nature of things and phenomena.

CHAPTER 5

The Taoist model. The theory of correspondence between minerals, plants and living creatures

From the furthest recesses of time, human beings discovered that specific plants and minerals could heal illnesses, but they were mostly limited to memorising which plants gave positive results. The ancient Chinese shamans instead developed a system that made it possible to identify illnesses and their cures in an "automatic" manner.

The Chinese were convinced they had a system which could indicate the plants that corresponded to a certain illness, and therefore heal it.

Initially, the indications obviously came from experience, but more than 2000, maybe even 3000, years ago, a classification system took shape that made it possible to understand which plant corresponded to a given illness. This was possible thanks to the cataloguing of the symptoms of the illnesses and the most evident characteristics of the plant.

The shaman translated the symptoms of the ill person into yin and yang, whole or broken line sequences, with the illness being identified, as such, by a binary number (2 paired hexagons).

The shaman then used a type of dictionary that listed the different sequences of whole and broken lines; this text indicated, for each sequence, a plant that had been classified using the same system on the basis of a series of facts: its flavour, the shape of its leaves, the size...

Basically, they gave a code number to the illness and then healed it using the solution pertinent to the corresponding code.

As traditional Chinese medicine continued, not only herbs but also minerals and animal parts were classified, together with

points of the body to be massaged, pricked with needles, heated with burning embers, or cupped[1]. Beyond the curious healing systems that were used in China, I find it fascinating to understand how this system, which coded states of health and the healing qualities of the solutions, were structured.

We are dealing with a kind of "universal translator" that the Chinese did not only use for healing but also for guiding their chemical research, with indisputably remarkable results.

It is interesting to note that this idea of "universal translator" contains the hope that we can understand the nature of creation. They were convinced that they could understand the nature of substances and illnesses only by observing some external, clearly identifiable, aspects. They believed that the human mind and our perceptual system, having developed in their relationship with reality, held the keys for understanding it.

The Taoist code for identifying reality

In the Taoist classification systems, illnesses and their cures are indicated by 2 hexagrams (therefore 12 whole or broken lines), in other words a "dictionary" is used that includes 4096 different codes (64x64), each one combined with an illness and a pertinent cure. Creating such a vast classification was certainly not an easy job!

The fulcrum of this system is that we are not facing a simple numeric code where each number is associated with an illness and a cure in a casual manner.

And the code number is attributed to an arithmetic and spatial (geometric) system in which the evident qualities specific to an entity determine the code.

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1 - Treatment involving the creation of a vacuum obtained by burning the air in a small cup that is placed on the skin, so that the vacuum produces a certain suction (cupping therapy).

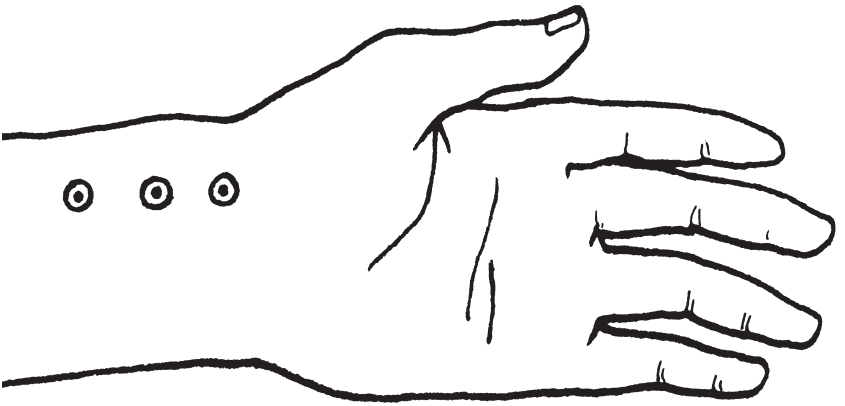
Diagnosis

Each line of each hexagram is associated with one of the 12 physiological functions (6 organs and 6 viscera which do not correspond exactly to our organs because some are grouped together). Various symptoms, both physical and relative to our mood, our way of moving and speaking, are associated with each function. One line is associated with the heart, another with the liver, another with the kidneys and they are fixed positions. If the shaman traced the line corresponding to the heart with a broken sign, it meant that yin was prevalent inside it. If the line was whole, this indicated a prevalence of yang.

Determination on whether each of the 12 lines, which represented the state of health of the 12 organs of a person, should be whole or broken was based on many precisely coded observations. They considered lots of aspects of the physical state of the patient: skin colour, muscular tone, nail and hair condition, smell, the quality of the faeces and many other things, too. They scrupulously noted these characteristics, determining if each symptom had a yin or yang character and decided, on this basis, what the dominating polarity of each one of the 12 functions was. After this, they traced 12 whole or broken lines.

They then used an additional verification system, which involved palpating the patient's pulse. They did, in fact, discover a particular anatomical aspect that is not considered by modern medicine. By placing three fingertips on a person's wrist, along the tendon, you can feel that the heartbeat in one or more points is different, either stronger or lighter *(Figure 16)*. By pressing, 3 beats with a different intensity than those of the surface can be heard. It is a curious phenomenon that can be easily verified, even without knowing anything about Chinese medicine.

The vibration of the pulse rate seems different to the touch if we place our fingertips at a distance of one centimetre from each other!



(Figure 16)

Each one of these 12 "pulses", 6 on each wrist, is associated with one of the 12 functions. The doctor knew how the beat should have been when the organ was healthy. He would notice if a beat was too strong or weak, interpreting these differences as an excess of yang or yin. This datum was very important for determining whether to record a whole or a broken line[2].

As already indicated, even the qualities of plants and their chemical substances were classified on the basis of their evident qualities, creating in this manner a code made of 12 whole or broken lines. To identify the qualities, a system was followed that considered multiple aspects.

For example, taste was classified.

According to the Taoists, there were 6 flavours:
Sweet Spicy
Sour

2 - Over the centuries, the auscultation of wrists became a continually more complex technique. Some medical schools classified "9 points" at each wrist and the many variants of the beat, which was considered not just strong or weak but also dry, resounding, slow, slippery, etc..., were taken into consideration.

Bitter
Salt
Umami[3]

The first 3 were flavours classified as yin, the second 3 as yang.

Even in this case there was a fixed relationship between each flavour and one of the 6 lines in the hexagram. If the line that corresponded to the bitter flavour was whole, it meant that the plant had a strong bitter taste, while if the line was broken, the taste was weak.

Even the choice of the point to prick, massage, warm or cup followed a rigorous method based on that same identification binary code.

The origin of the Taoist medicine system

Primitive Chinese medicine took its inspiration from the great economic importance of irrigation.

The Chinese had an excessive enthusiasm for water, channels and irrigation systems. It led them to develop colossal works such as the Jing-Hang Grand Canal, which they started to dig during the Shang dynasty, more than 3000 years ago. It was a work that continued for 2000 years and which goes far beyond the construction of the Egyptian pyramids and the Great Wall.

The centrality of irrigation in the Chinese economy led them to use a water system as a model for the human body: they im-

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3 - I couldn't find a detailed catalogue of the plants. They obtained different classifications of flavours, the most common one being based on 7 flavours. In symmetry with the diagnosis using 6 pulses/hexagrams, I hypothesise that in ancient times there was a catalogue of 6 flavours, which seem to be evident to me: sweet, spicy, sour, bitter, salt, umami.
The first were yin flavours, the second were yang. Umami is the name used for the sixth flavour, the word is of Japanese origin. In European cuisine, Parmesan, truffles and snails are umami, but our culture does not identify this flavour. In the 1980s, it was shown that the tongue really does have taste receptors specialised in perceiving this aroma, which demonstrates how well the Chinese observe.

agined that the body was governed by a system of channels in which vital energy flowed (with yin or yang "polarity"). The Chinese model counts 12 energy channels that rise from the hands and feet towards the torso and head. Each channel is associated with an organ or one of the viscera.

The illness presents itself as a malfunction in the circulation of Qi (or Ki, Chi or Ci) along the channels, in the organs and in the viscera. Simplifying, we can say that in the same manner as plants that die if they receive no or a scarce quantity of water, organs suffer if they have too much or not enough life energy. Along each channel there is a series of points that operate like a type of tap or which are close[4]. These taps can be used to regulate energy circulation, re-establishing equilibrium.

Obtaining the identification code made up of 2 hexagrams (12 lines) from observation of the patient's physical and physiological state and the state of his pulses, it was possible to identify the exact point/tap it was best to work on and the most suitable action. It was not an arbitrary decision, but the result of a series of calculations which, starting from the code relative to the pathology, allowed them to reach the code that identified the point/tap to be treated and also the most suitable type of healing solution from among the 6 possible ones: pricking with a needle, massaging in depth, massaging on the surface, heating, cupping, using a compress.

Summing up: the shaman evaluated the state of the 12 functions

4 - For decades the existence of these channels and energy points was challenged by Western science, but during these last years new technology has made it possible to photograph the points. On the fact that they have a particular anatomic structure there are now no further doubts (see the studies of Gabriele Saudelli on the subject, published by the Italian Academy of Traditional Chinese Medicine). This obviously does not mean that the theory built on these points and the healing methods based on intervening on them truly work. It is interesting to note, however, that the ancient Chinese mapped anatomic points which, even though they truly exist, were not identified in the West.

of the patient's body and identified the illness with a code. He then used this code to select the channel and point of the channel on which to act, and the most suitable action at each point. He used the same code also to identify the plant or chemical substance that corresponded to the illness and which was suitable for healing it[5].

So there is no magic in ancient Chinese Traditional Medicine. There is only the presumption of becoming familiar with a system that puts a specific illness on the same plane as a special plant or point of the body. That was no small thing for 3000 years ago!

I observed that the interest in this system began from some initial discoveries on the aspects of reality, not from mystical magicians. The same thing is also true for arithmetic geometry: if you live in an era when eating every day is difficult, the reason that pushes you to waste time on numbers or the sounds of a pulse at the wrist must be truly fascinating. There has to be a simple initial discovery that fills you with enthusiasm and motivates you.

The birth of Chinese medicine was stimulated by the discovery that dealing with a particular point of the foot could relieve headache, therefore the discovery of reflexology points. It is said that the casual discovery of one of these points was fundamental for Traditional Chinese Medicine. A prince afflicted by a terrible headache decided to go hunting to distract himself from the pain. An archer shot an arrow which, by mistake, struck the

5 - Over the centuries, traditional Chinese medicine has developed greatly with the discovery of many new points, losing in this manner the perfect correspondence between points to be treated and identification code. Even the diagnosis system and choice of the cure was extended, and many traditional medicines have abandoned the calculation system used to identify pathologies and cures. However, the points that were considered in ancient times can still be identified today because they preserve in their ideogram a sign that identifies them as "ancient". There are 5 ancient points on each of the 12 channels plus one point, called congenital Qi, pertinent to an internal organ and the organ coupled with it. In the primitive model, therefore, 6 points were considered for each organ or an internal organ, which permitted correspondence with the model based on 2 hexagrams, 6 pulses on each forearm, 6 organs and 6 internal organs, etc. I have written a book called *The numeric structure of Primitive Traditional Chinese Medicine on this subject.*

prince's heel, making him happy because his headache passed immediately. It was therefore thought that the pain was caused by a demon hidden in the foot, and that the arrow had killed it. In this manner, the Chinese began mapping the points/taps.

The subsequent discovery was the different perception we have of the reflex of the pulse rate listened to at 3 points in the wrist.

Hence, the relationship between the intensity of the beat felt and the health state of organs classified as corresponding to each individual "beat". It would have been an insignificant discovery if it had not been confirmed by a simple empirical observation. If one of the 12 pulses is visibly non-harmonious as to the others, it is almost certain that the 6 Ancient Points along the corresponding energy channel are painful. It is a curious correspondence.

Obviously, this does not mean to say that the theory as a whole is correct, and it does not mean that the diagnosis is correct, nor that the cures theorised by Chinese Medicine are effective, but this correspondence at least has great power in influencing the ill person.

The doctor, without knowing the patient, without asking what symptoms he had, touched his pulses for a few minutes and then said: "I will now gently press a point that will hurt a lot." And when he pressed, it really did hurt, exactly there and nowhere else, an acute, unusual sensation.

This astutely ironic procedure gave ill people the idea that the doctor "knew". At that point, the doctor had certainly managed to mass mobilise the placebo effect…[6] He maybe then correctly found a reflex point that did actually soothe the pain. The techniques of Chinese anaesthetics certainly have a fundamental

6 - At the Libera Università di Alcatraz we have, for decades, been discovering the tricks used by particularly astute shamans and gurus. We carried out an experiment with lots of people on the effective correspondence between non-harmonious pulses and their corresponding points. We verified that after a few hours of lesson, all the students were able to recognise the same non-harmonic pulses, to identify the same corresponding points to be treated, always finding that the patient felt pain if gentle pressure was applied to that point.

reality, so much so that today they are used even in modern hospitals for people who cannot support synthetic pain killers.
I am not interested in discussing here the effectiveness or not of the therapy system from a scientific standpoint. If it continued being used for thousands of years, this primitive cure certainly had to give positive results. But, as I highlighted previously, maybe it was effective only as a refined placebo.
What interests me is describing the arithmetic and geometric structure of this identification system, because it contains some interesting discoveries.

CHAPTER 6

Geometric and spatial arithmetic

As mentioned previously, the Kabbalist and Taoist models contain a very particular idea on the possibility of using numbers to indicate not just quantity but also quality. We can define this idea as spatial arithmetic.

In the system with the 8 trigraphs, each element of the sequence contains an additional piece of information that is determined by its form.

It is a formidable cataloguing system because it supplies more information than normal binary numeration.

The symbols shown above contain two whole and four broken lines, therefore they are equivalent, but it must be considered that the different layout of the lines has a meaning. As already seen, this was verified in Traditional Chinese Medicine because a different physiological function was associated with each position. The upper trigraph indicated 3 organs, and the corresponding 3 points where the wrist was to be palpated. The second trigraph was connected to the three internal organs coupled with the three organs, which could be auscultated by pressing the three palpation points on the wrist more.

Over time, the forms that were obtained started being considered as important, too. As an example, it was considered that a state of health represented by two hexagrams with the same

number of yin and yang lines positioned symmetrically was more positive than a state of health that was just as balanced in yin and yang lines but positioned in an asymmetrical manner. In other words, the doctor obtained information about the illness and the cure that went beyond simple cataloguing of the symptoms, because the construction of the two hexagrams that indicated the state of health of a specific patient supplied an additional element: the symmetry or asymmetry between the two hexagrams that represented the diagnosis obtained from listening to the two wrists.

To better clarify this hypothesis on how the idea of numbers containing a piece of information that was not just quantitative could be formed, we have to return to what was said initially about triangles.

Numbers that are shapes and ideas

As mentioned before, triangles probably played an important role in the birth of enthousiasm for geometric forms and numbers.

Ancient wise men observed that the idea of 1-2-3 had a certain similarity to the shape of triangles: the first elementary observation was that a triangle is made up of three lines.

They then realised, however, that there is a second quality based on 3: it is formed of 3 elements: area, sides and corners. The triangle was therefore made up of 3 elements from both a geometric (3 lines) and a conceptual (its components could be classified into 3 categories) standpoint.

If a mathematician today has to use a symbol to indicate the series of corners, sides and area of a triangle, he would probably label each element indifferently with a number from 1 to 3. He would certainly have no reason to associate the 1, the 2 or the 3 with the group of sides. The numbers would be used as neutral symbols.

Our ancient researchers decided, instead, to take a chance:

they tried to indicate each type of component of the triangle with a number that would be logically similar to the entity it represented. In other words, they tried to match forms, quantities and qualities, inventing a concept that is a hybrid between number, shape and idea. Put differently, they gave a meaning to the position of the number inside the model. The concept of area resonated well with the concept of uniqueness, because it is a unique set. Area was catalogued as an element belonging to the same category as the number 1; lines had a starting and an end point, therefore they were intrinsically dual and were associated with the number 2; corners were composed of 2 elements (the sides) that generated a meeting point, the corner, therefore they were composed of 3 elements and were associated with 3.

By reasoning in this manner, numbers that had not only a quantitative meaning were obtained, because each one was associated with one of the components of the triangle.

But the ancient wise men knew how to go beyond this.

For us, the numbers 1, 2 and 3 are like all the others. They have special properties, of course, but they don't have a meaning.

The ancient wise men noted, instead, that 1, 2 and 3 contain 3 concepts.

Firstly, they are the only numbers that are present everywhere. I know it seems stupid to say such a thing, but from a certain point of view the evidence is undeniable: each thing is its own entity therefore it contains the quality of 1; each thing is the result of the interaction between the 2 elements of a contradiction (we are male or female, time is divided into day and night, we find phenomena based on the contrast of two antagonistic and complementary powers, heat and cold, action and reaction); each material entity has 3 dimensions (height, length, depth), can have 3 different consistencies (liquid, solid or gas), and can be present in the past, the present or the future, a man and a

woman make love to generate a creature, 2 warriors that crash create a fight. And it should be noted that the concept 3 contains 1 and 2 even from a logical standpoint.

This is how the ancients managed to give a meaning to the first 3 numbers that went beyond quantitative information. From there, they started developing their fractal model of the world.

An interesting aspect is that these original researchers did not go any further, they were modest. They were certain of these 3 qualities. They did not know much about the rest, so they developed their thought by extending observations on only 3 elements that they believed to perceive clearly.

CHAPTER 7

Did even Kabbalists build a numeric and geometric model of reality?

Up until this point I have tried to describe the Taoist model of the world. I abandoned Pythagoras and the Mediterranean wise men because their knowledge was mostly lost.

Now, we can gather the little evidence available on Western ideas and see if it matches the Chinese model, and so launch hypotheses on how the whole pot could have been. In this manner, we will also see that we are dealing with 2 models that follow the same procedure and reach the same conclusions, the differences are only apparent.

We can obtain some information on the Western model from the Kabbalist tradition, the only one that hands on fragments of the numeric model of the world developed in the West and starting from Pythagoras.

The Kabbalah (or Kabbala, Kabalà, Cabbala, Cabalà, Cabbalah) is a complex system for interpreting the esoteric meanings of the Bible. It is based on the fact that each letter of the Jewish alphabet has a symbolic meaning that can be understood if the letters are transformed into numbers. The Jewish alphabet includes 22 letters and this number is central also to the Mediterranean system, not just for Jews.

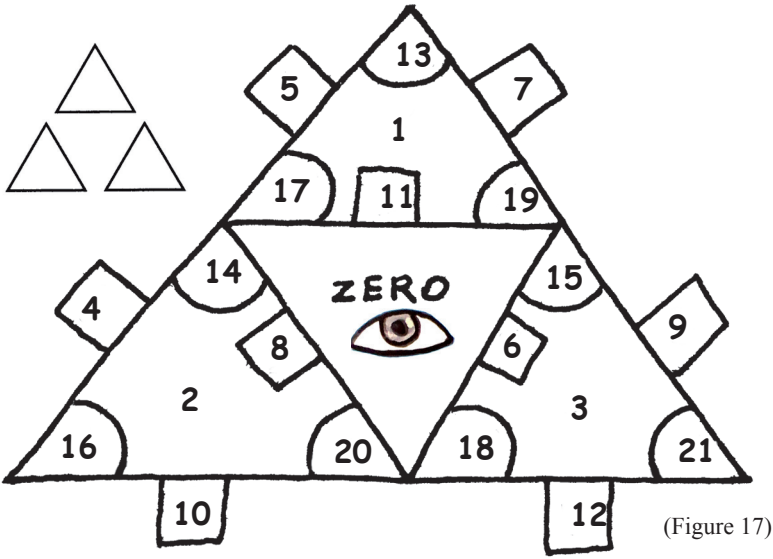
There are 22 tarot cards, which give a meaning to each number. Tarot cards are the Christian version of the Jewish Kabbalah.

The first characteristic of this system is that it is based on 22, a number that was not selected casually. It comes from the fact that the Jewish alphabet has 22 letters, but a careful study based on this number was made that went beyond this correspondence.

Indeed, 22 emerges spontaneously from the grid of triangles that we described at the start of this book.

As I have already said, placing 3 triangles next to each other creates a fourth that was not drawn, and this was considered a phenomenon that represented, in the structure of shapes, the relationship between God the creator and creation, a perfect representation of the concept of divine trinity that was, as such, adopted by Catholics as a symbol of the divine: the eye of God was drawn in the centre of an equilateral triangle.

My hypothesis is that they dedicated a lot of time to observing this divine symbol, and it probably didn't take them long to realise that this diagram contains the number 22: in fact, we can count 1 area + 3 sides + 3 corners for each of the 3 "real" triangles = 7x3 = 21 + the zero, which represents the central, virtual triangle = 22 *(Figure 17)*.



(Figure 17)

And this was proof of the fact that 22 was the unit of measurement of the world. It also suggested the idea that each phenomenon was divisible into 22 constituent elements, in the same manner as the alphabet that included 22 letters.

The wise men asked themselves if this number could be used to build a structure for reading the ingredients of reality.

It was an idea that gave rise to a knowledge system that was "magical", oracular, which is beyond the theme of this work. But there is an aspect in the construction of the Jewish divination system that is interesting to note.

The kabbalists convinced themselves that everything could be divided into 22 basic elements.

For them, even the events of human life could be classified in 22 categories, 22 phases. Life was a kind of game, like snakes and ladders, and what happens while you are in a square is characterised by the quality of the square itself.

As such, the idea of drawing signs using the letters of the alphabet as symbols arose. With tarot cards, these phases of life are illustrated by images that represent existential archetypes. The Lovers card represents a phase in which a choice has to be made, Death represents a moment of drastic change, Stars show rebirth, and so forth. The idea is that a human being can only pass through 22 phases. Obviously, it is a schematization of the complexity of life that each one of us goes through, but to understand how the ancients thought, it is important to consider their thirst for simplicity, their desire to be able to reduce the dizzy and disturbing reality of the world into a simple outline.

It is also interesting to note, though, that this way of reasoning implies an idea that goes beyond mystical beliefs. The letters of the Jewish alphabet and tarot cards take their meanings from being numbers and concepts at the same time. Each meaning descends from the nature of the first 3 numbers that are the matrix of everything. In other words, the meanings of the Jewish letters are not assigned following mystical suggestions but on the basis of their numeric value. Each letter has its own position, and as such a number. In our alphabet, the letter A is always first, and is always followed by B and then C.

They therefore correspond to 1, 2, 3.

When they gave a significance to the first three letters of their alphabet, the Jews gave each letter the meaning of the corresponding number.

One is the smallest whole number, so it contains the concept of uniqueness and also of the first existing entity. The number one gives rise to everything, therefore it represents the creative force.

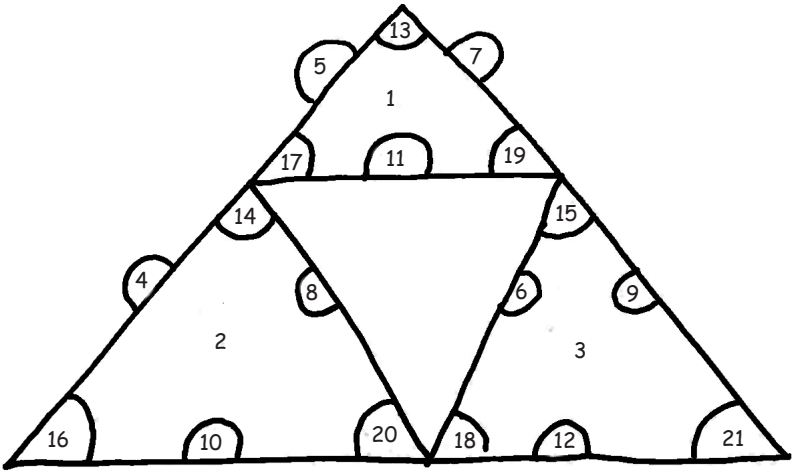
Two is the first pair, one and two give rise to the concept of derivation and enumeration.

Three represents the couple that generates a child, but also the ternary form of the idea of time: past, present and future.

I also imagine that the ancient kabbalists became conscious of a detail about numbers that I have not seen mentioned anywhere, but which corresponds perfectly to their ternary model: numbers can be divided equally into 3 groups: there are odd numbers that are not divisible by 3, numbers that are divisible by two but not by three; numbers that are divisible by 3.

And I believe the ancient wise men noted that in the numbers from 1 to 21 there are 7 numbers that belong to each of the 3 groups. If we add zero to this, we obtain the number 22. And, as a matter of fact, the Kabbalah considers 22 as the result of 21+zero. If we place these numbers in columns by group, we obtain this result:

A	AREA	1	AREA	2	AREA	3
B	SIDE	5	SIDE	4	SIDE	6
	SIDE	7	SIDE	8	SIDE	9
	SIDE	11	SIDE	10	SIDE	12
C	CORNER	13	CORNER	14	CORNER	15
	CORNER	17	CORNER	16	CORNER	18
	CORNER	19	CORNER	20	CORNER	21
ZERO						



I can use this diagram as an outline to select the numbers to be combined with the 21 components of the 3 triangles that are next to each other: this is the interpretative plan of the 22 numbers of the Kabbalah. This plan also perfectly superimposes the geometric model of the 3 triangles that generate a fourth.

At this point, my reconstruction of the Kabbalist model stops.

All the reflections that the Mediterranean ancients surely made are lost, burnt, most of the wise men killed.

Maybe one day precious parchments will appear, purloined from the library in Alexandria, and we will discover that a system similar to the Chinese one had been created, maybe a ternary numeration that transforms the 21 numbers into sequences of 1, 2 and 3.

Or we will discover that even the Rabbis had idealised a code that classified the qualities of plants and minerals. Who knows what was written in the casket of notes that Newton burned and on Leonardo's lost sheets…

CHAPTER 8

Kabbalists and Taoists: a single model, apparently different

At first sight it seems that, starting from the first 3 numbers, Jews and Chinese obtained two very different models. It is, however, the same identical model but described under a different aspect: components or movement. The Kabbalists favoured the geometric aspects of the system and in particular they broke up the forms into their components (sides, corners, areas). The Taoists instead preferred the numeric aspects of the phases of movement, and centred their version of the same model on 64.

The reason for this difference is of linguistic origin. Mediterranean idioms are made up of distinct sounds that can be easily identified with a limited number of characters.

The Chinese could not follow this phonetic road because their language is based on tone in addition to sound. This means that a word with the same sound can have up to 8 different meanings according to the intonation used. As a result, the Chinese chose not to write words by dividing them into sounds, but to summarise the meaning of the word in an image, an ideogram. The model passed on by the Jews and the one handed on by the Taoists are apparently different because they depend on two different procedures. The Mediterranean populations tend to classify the components, while the Chinese tend to describe them together, in their action. At this point I suspect that the association of both aspects to 22 and 64 is not artificial, but emerges spontaneously from observation.

Firstly, we can observe that the general layout of both models is identical. Both start from the graphic representation of numbers that are forms and concepts (unit, pair, tripartition). Both hypothesise the existence of a single basic entity that gives sub-

stance to everything that exists. This entity is incorporeal and cannot be classified[1]. At a certain point, this entity acquires two polarities, therefore each polarity differentiates again, giving rise to 4 primitive forms of the same basic entity (I will deal with the Taoist vision shortly).

From this point onwards, the two models apparently become different: the Taoists identify 3 types of fundamental brick, and so first develop 8, then 64, then 64x64 entities created from trigraph pairing.

The Mediterranean has the model based on 21+1=22.

It is true that even in the Western model of geometric arithmetic the numbers 8, 16, 64 emerge more than once, but the resemblance is rather small.

The correspondence between the two approaches emerges domineeringly when the 64 Taoist hexagrams are observed, considering them in a similar manner as the Pythagorean figurate numbers.

From this standpoint we can say that the 64 hexagrams are effectively the figurate representation of the binary numbers, namely Pythagoras invented decimal figurate numbers, the Taoists binary figurate numbers. And if we try to observe this binary numbering in a similar manner as that used to observe the decimal figurate numbers, there is a surprise in store.

The layout of the whole and broken lines of the hexagrams makes it possible to divide them into categories following two different procedures based on the observation of two different orders of arithmetic geometry evidence.

The first evident observation is that the 64 hexagrams can be divided into 2 groups.

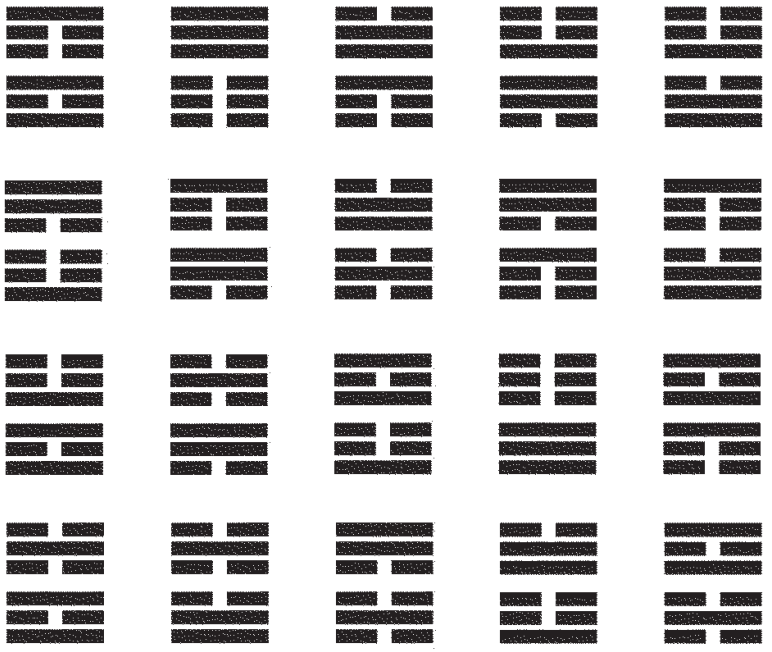
By observing the layout of the whole and broken lines that make them up, we can see that 20 are formed of an equal number of whole and broken lines, therefore they have a balanced

1 - See Il Tao della fisica (The Tao of physics) by Fritjof Capra, adelphi, dedicated to the relation of the ancient visions of matter.

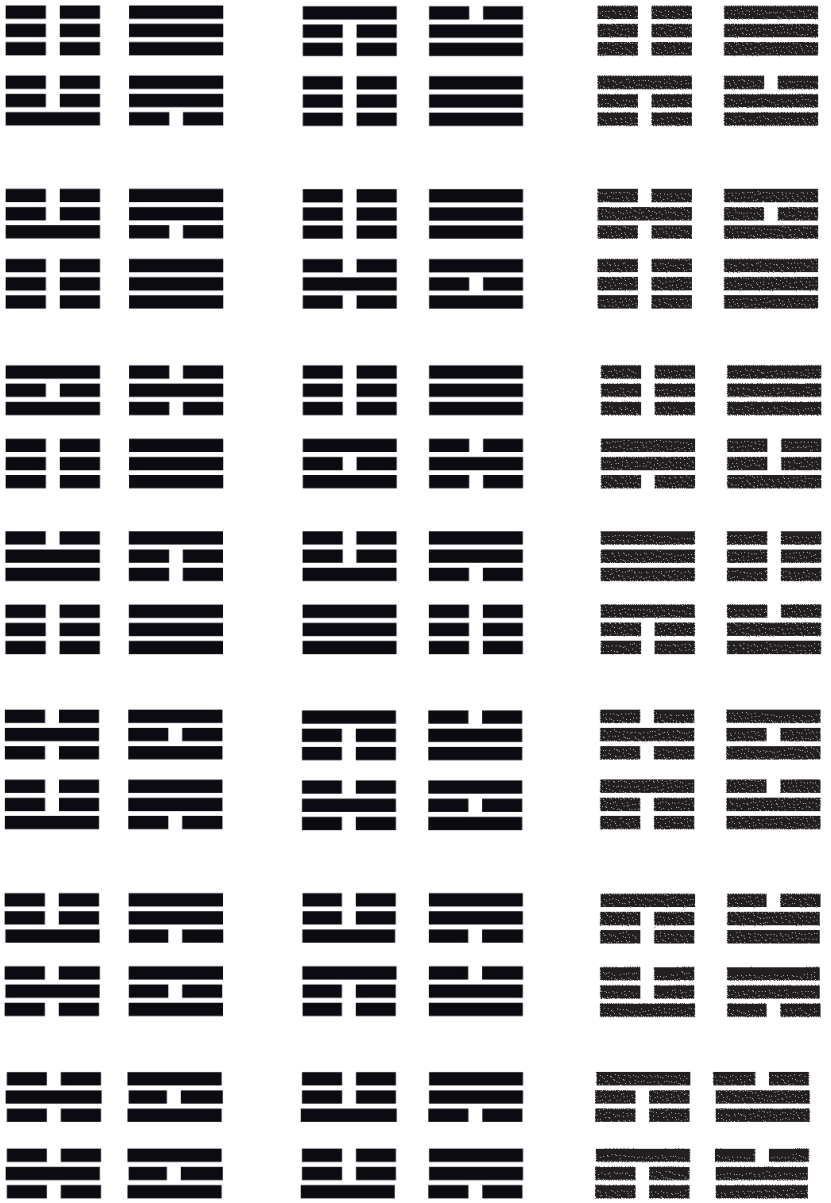
composition, and 44 by an unequal number of whole and broken lines. Two of these 44 hexagrams are structurally different though. One has 6 whole lines and one has 6 broken lines, and they are special because they cannot be associated with any state of health of a living being. If all the organs of an ill person are too full or too empty of energy, that person is dead (life is the effect of the synergy between the 2 polarities).

The 44 hexagrams are therefore conceptually 42 real + 2 only virtual. And so, the Kabbalist number of 21+1, repeated twice, emerges from the form of the hexagrams.

20 balanced hexagrams



42 unbalanced hexagrams

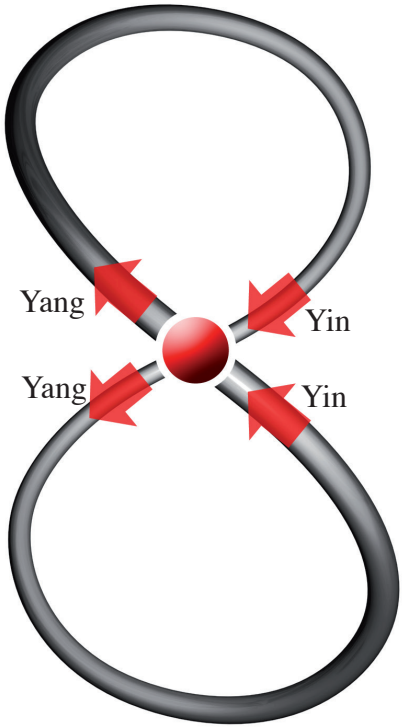


2 virtual hexagrams

This division of the 64 hexagrams into 3 groups is correlated by the Chinese idea of the origin of matter. At the beginning, there was only one form of energy: Tao, the creative nothing.

Tao expands and acquires 2 polarities moving in two directions. As a result, yin and yang, two energy states, exist.

The particles that move from the centre towards the surface are yang, and the particles that from the periphery return towards the centre are yin. Each particle, according to the segment of route it is following, is characterised by one or the other polarity. Each particle is initially yang, then becomes yin, then returns being yang, and so forth. The centre of this movement is extremely crowded, the particles collide, and many are thrown out of their route.



Because of these collisions, a part of the yin and yang particles acquire a rotary movement around their own axis, so 4 phases of particle movement exist.

One section of the particles therefore moves in a spiral manner, which can be clockwise or anticlockwise, giving the 8 phases of particle move-

ment. We therefore obtain 3 different qualities of the same polarity according to the movement phase. Each particle, as it continues along its path from and towards the centre, crosses 8 phases.

The three qualities are represented through the 8 trigraphs that indicate that phase in which each fraction of the Tao is found.

At this point, the initial balance breaks, explodes, and the particles couple to form 20 pairs represented by hexagrams that are balanced because they contain particles which, at the moment of explosion, were in opposing phases[2].

The remaining 44 pairs of particles do not contain enough yin and yang, so they do not have enough cohesion to remain united for long, and they unite with other pairs of unbalanced particles.

22 pairs originate, represented by pairs of hexagrams. One of these pairs is only virtual because formed of 2 hexagrams of all whole lines and all broken lines.

This passage closes the first phase of the birth of the universe, from which 5 types of entity generated by Tao spring:

- Nuclei of energy in the yin and yang phase
- Nuclei of energy also with spin movement
- Trigraph particles
- Hexagram particles
- Pairs of particles (double hexagram)

These are the 5 phases of mutation[3].

Finito questo primo ciclo il procedimento si ripete e una parte delle coppie di esagrammi/particelle si uniscono creando combinazioni sempre più complesse e dando così forma e qualità a tutte le sostanze chimiche. Poi nuovamente il ciclo delle 5 fasi del mutamento si ripete dando origine alle cellule… E via così.

---

2 - A particle that rises with a rotary movement and anticlockwise spiral pairs with another that falls with the other two anticlockwise movements.

3 - They are also known as The 5 elements but the term “elements” is misleading because we are speaking about elements that are part of a process of becoming. We are, in reality, dealing with movement phases, indeed classic texts specify that each element develops into the next one in a wheel of becoming.

Da questa esposizione risulta evidente che il sistema dei 64 esagrammi contiene strutturalmente il 21+zero identificato come una costante quantitativa delle componenti di un'entità (le coppie di esagrammi squilibrati).

Inoltre le 5 fasi del mutamento hanno la creazione dei 21 aggregati di particelle come culmine del loro processo evolutivo.

È elemento essenziale che i taoisti identifichino in 21 le tipologie di particelle possibili, perché dimostra la corrispondenza concettuale con il modello cabalista che identifica in 21 + zero le componenti della realtà (nell'alfabeto una lettera è muta).

Twins Hexagram

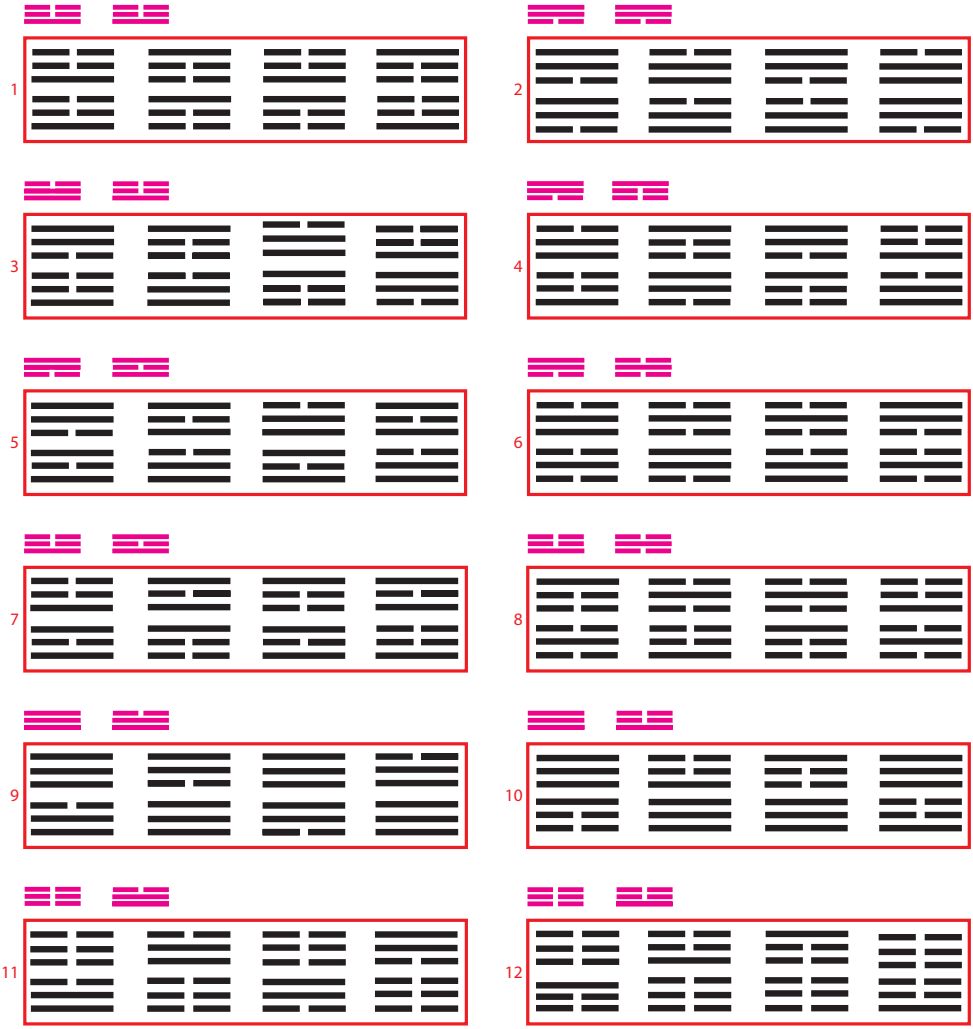
There is another way of dividing the 64 hexagrams into categories, on the basis of the fact that the majority of the hexagrams has an equal but upside down twin.

We have said that the hexagrams are made up of 2 trigraphs. In 12 cases, those 2 trigraphs can be paired in 4 different ways, giving 48 trigraphs.

There are also 12 trigraphs that are equal, 2 by 2, so making 6 pairs. Finally, 4 hexagrams do not have synonyms.

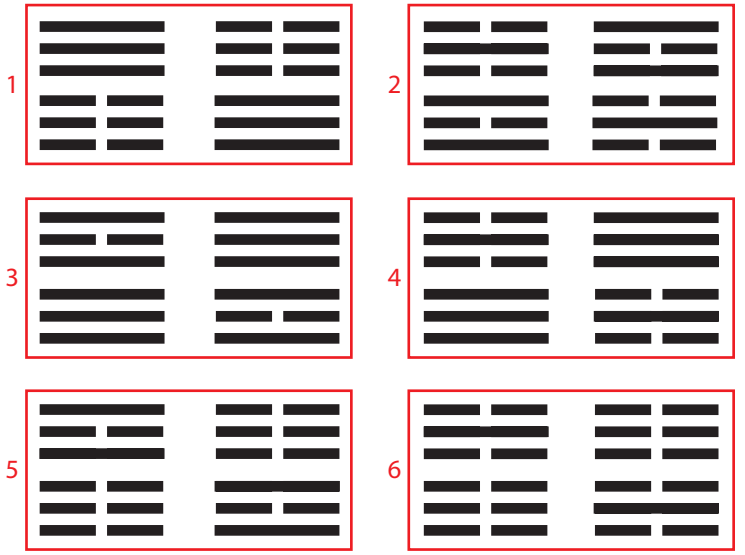
Two of these are only virtual and represent zero (a hexagram with only whole lines and one with only broken lines). Once again, the 21+zero of the Kabbalah emerges.

12 groups of 4 equivalent hexagrams (made up of two equal trigraphs in an upside-down or specular position) *(Figure 18)*.



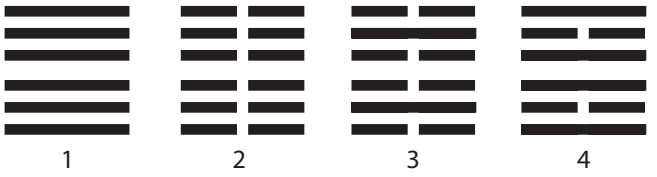
(Figure 18)

6 pairs of equivalent hexagrams (Figure 19).



(Figure 19)

4 hexagrams without an equivalent hexagram *(Figure 20)*.



(Figure 20)

This classification has a strong analogy with the division into groups of DNA language.

It is made up of 64 triplets, or codons, each one of which includes 3 nucleotides. Each codon corresponds to an amino acid. As 20 amino acids are involved in the formation of proteins, they

are generally coded by more than one codon. There are, in fact, groups of 6, 4, 3 or 2 equivalent codons for creating a signifier.

Three of the 64 codons do not correspond to amino acids, instead operating as translation termination signals (UAA, UAG, UGA stop codons).

The AUG codon, which codes for methionine, is also the start codon, the signal that starts translation.

The 64 codons therefore have 22 meanings.

CHAPTER 9

What value does this fractal model have?

The first observation we can make regarding this forgotten theory is that it adheres perfectly to our system of perception.

Primitive researchers have the merit of following the only evidence of reality, limiting themselves to classifying the aspects that human senses can identify in a unanimously shared manner. They did not invent anything. Primitives managed to trace a map of the way in which our mind perceives some aspects of reality, geometry and arithmetic.

So, this model of the world should be considered at least from the standpoint of the frame of perceptions that it describes. If the deduction is that it does not correspond at all to reality but only to the manner of perceiving it, we would be facing an attempt to draw the impression that reality has stamped in the cognitive structures of our mind. It would be an interesting result from the standpoint of the study of perception, and also from a historical one.

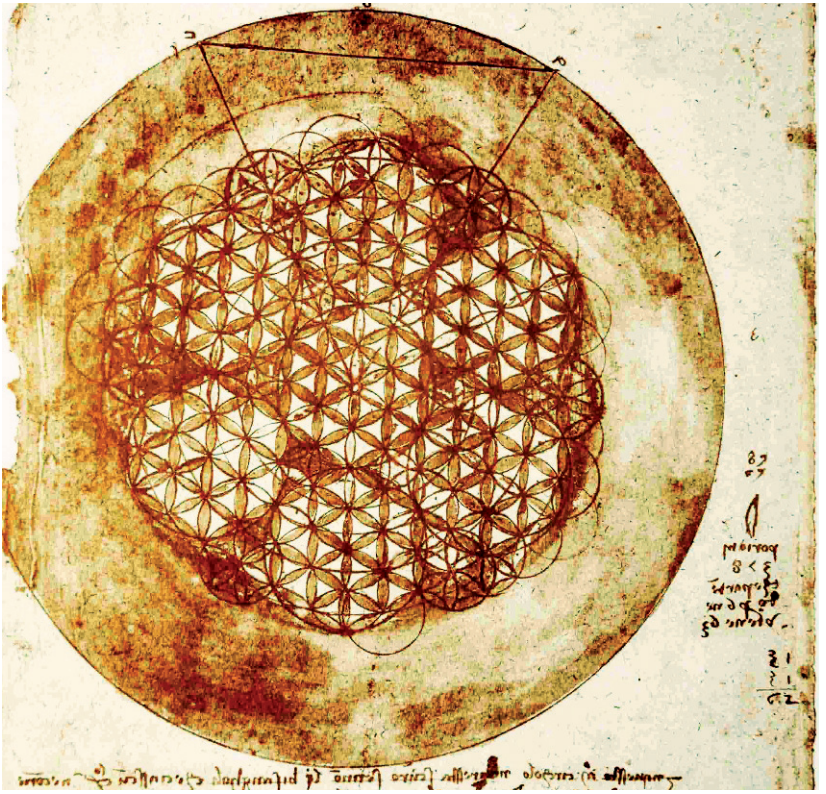
I suspect it may also contain observations on reality that could offer interesting suggestions that I consider wasted if not looked further into.

Evidently, the diffidence of scientists towards magic, the occult and religious dogma ensures that the whole matter is liquidated as mystic junk without interest. It is forgotten that this model was at the centre of the reflections of some of the greatest scientists from the past.

When the majority of scientists speak about Newton, they cleanly divide his scientific observations from his colossal research on shapes, dismissing the latter as the consequence of

a time dominated by superstition. Newton's work is considered

as being divided by a gulf. There is Newton the mystical alchemist, and also Newton the scientific researcher. Could it not be that the study he developed on how a grid of triangles generates numbers and forms helped him in his scientific research?



Drawing by Leonardo da Vinci: the grid of equilateral triangles generates the Flower of Life.

But does it work in reality?

Having said this, we can ask ourselves if this idea has a correspondence outside the world of perceptions: does concrete reality give confirmation?

Is this fractal model only an artificial construction, or does it contain the description of a real aspect of the relationship between sound, numbers and geometric shapes, not greatly considered by modern science?

Is the relationship between 1-2-3, 6 and 8, 22-64 an objective phenomenon similar to the Fibonacci sequence? Does the fact that this sequence emerges from a grid of triangles prove that this constant permeates reality in a fractal manner?

To cultivate a reasonable suspect in this direction we can, first of all, observe that the arithmetic geometry model of the ancients highlights a series of coherent relations between some numbers and some forms. This seems quite strange to me.

I list below the aspects we have gradually identified in the previous pages, adding some observations.

The numbers 1, 2, 3 have unique characteristics from an arithmetic standpoint: the sum of 1+2+3 gives 6 and multiplying 1x2x3 still gives 6.

The numbers 1, 2, 3 correspond to the uniqueness, duality and tripartition of everything that exists. These 3 qualities are evident in numeration; all the numbers are uniqueness, they are even or odd and they can be divided into 3 categories: odd numbers that are not divisible by 3, even numbers that are not divisible by 3, numbers that are divisible by 3.

Observing a grid of triangles, I note that increasingly more complex forms emerge: a hexagon can be seen in a grid made up of 9 triangles (the Tetraktys of Pythagoras).

If I draw a grid of hexagons above a spherical surface, a pentagon emerges.

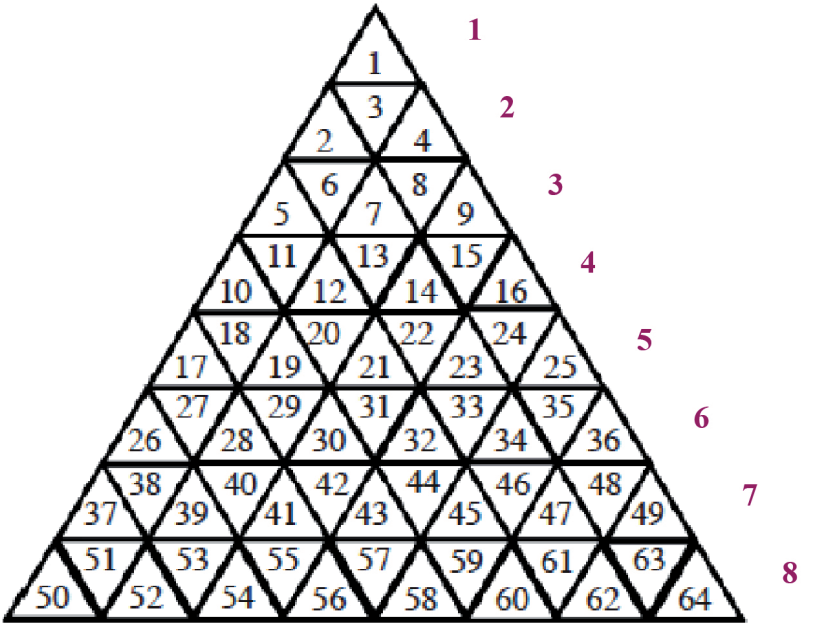
If I draw a grid of pentagons on a flat surface, a 5-pointed star that contains Fibonacci's proportion ratio appears.

Looking at a hexagon inscribed in a grid of triangles, I can discern a cube, which has 8 vertices and can seem to be either concave or convex.

I can nominate the 9 vertices of the cube using a 3-figure binary number.

Eight different forms emerge from a grid of triangles.

A grid of triangles that contains 8 triangles vertically is formed of 64 triangles.



Grid with 8 levels, contains 64 triangles.

If I draw circles which have the crossing points of the grid as their centres, I obtain the Flower of Life.

This image can also be seen as the image of 8 spheres inscribed in a cube.

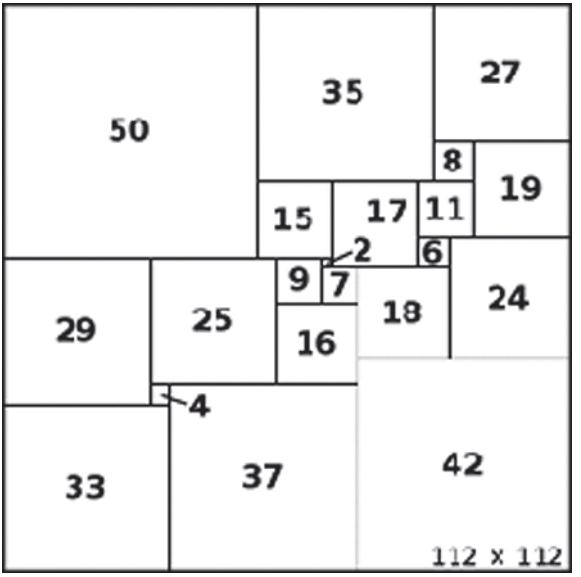
If I draw a bigger Flowers of Life grid I obtain a cube made of 64 spheres.

The sum of the numbers 1-2-3-4-5-6 is 21, and by adding zero we reach 22.

If I place 3 triangles close to each other they form a fourth, and I can indicate the geometric components that make them up with 21 numbers + zero.

My Polish friend Davide Staunovo, who had the patience to follow the development of this work, indicated that the concept of 22 as 3x7 can also recall all the possible notes of 3 musical octaves (in fact 3 octaves are not 24 but 22 notes, because the central Cs are not repeated).

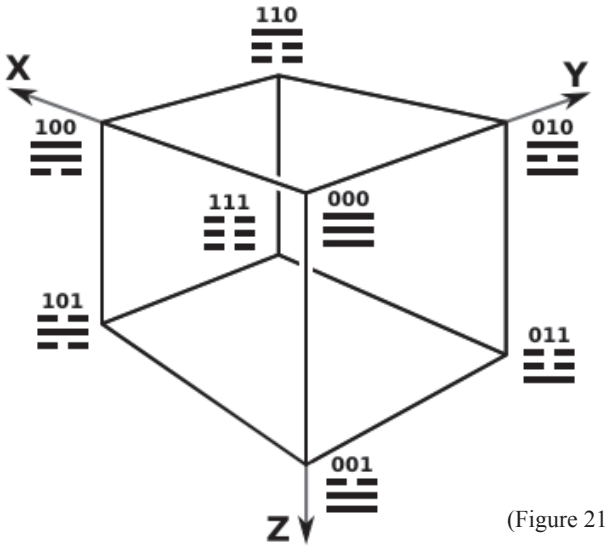
The extremely kind Edoardo Pedio pointed out to me that the smallest number of different squares that in turn form a square is 21, you need 21 squares to fill a square.



The 64 hexagrams are divided into 20 balanced hexagrams and 44 non-balanced ones which, combining with each other, form 21 pairs + 1 virtual.

If I divide the 64 hexagrams on the basis of their layout and the number of yin and yang elements that make them up, I can see that many hexagrams are equivalent to each other and I obtain 22 different classifications in this manner, of which 1 is only virtual.

Finally, we can observe that the 64 hexagrams correspond to a Cartesian coordinate system referring to a cube (see Figure 21).



(Figure 21)

If I refer the figurate numbers to the vertices of a grid of triangles, the number of triangles contained in each triangular number is a square number.

An ancient modern idea

Another aspect worth noting lies in the general idea that emerges from this model.

Today, we hear about the physics of informed matter.

We begin suspecting that the constituent nature of the energy that gives mass and movement to particles has more to do with the ability to behave in a certain manner than with the existence of solid matter; in other words, the primary component of solid matter is not a physical substance but the information that makes it possible to generate it.

It was the ancient Taoists who spoke about information.

The quality of Yin and Yang does not lie in physical mass, which they do not have.

From this point of view the most important characteristic of the 64-hexagram system is that the hexagrams represent a type of numbering that does not only contain the arithmetic datum, it also allows us to consider the spatial layout of the whole and broken lines that make up the hexagram and which therefore contain information.

We are facing a conception of the number which, having an "important" spatial form and a correspondence with geometric constructions, contains information on number quality that the numbering used today cannot represent.

Another thing piques my curiosity: the fact that this model contains a fractal idea of the world and codifies the development of this fractal in the 5 phases of mutation.

Just as amazing is that observing this model highlights a vision in certain ways similar to the quantic idea of reality. Indeed, one of the fundamental aspects of the quantum model is the observation of indeterminate states.

It is exactly what happens in the Taoist/ Kabbalist model.

If observed, it can be seen that an entity is made of 22 components, and that it has 64 movement phases.

This impossibility of contemporaneously taking 2 aspects of reality has generated apparent differences between the Chinese model and the Jewish one.

This vagueness reappears several times in the model of the ancients.

The hexagon in the Tetraktys by Pythagoras is a three-dimensional cube that can appear to be concave or convex, according to how you look at it.

As soon as I place 3 triangles next to each other in a certain manner, a fourth one appears from nowhere. If I name the areas, sides and corners of the 3 triangles placed next to each other, I obtain 21 entities, and if to these I add the area of the virtual triangle in the centre, I obtain 22 elements.

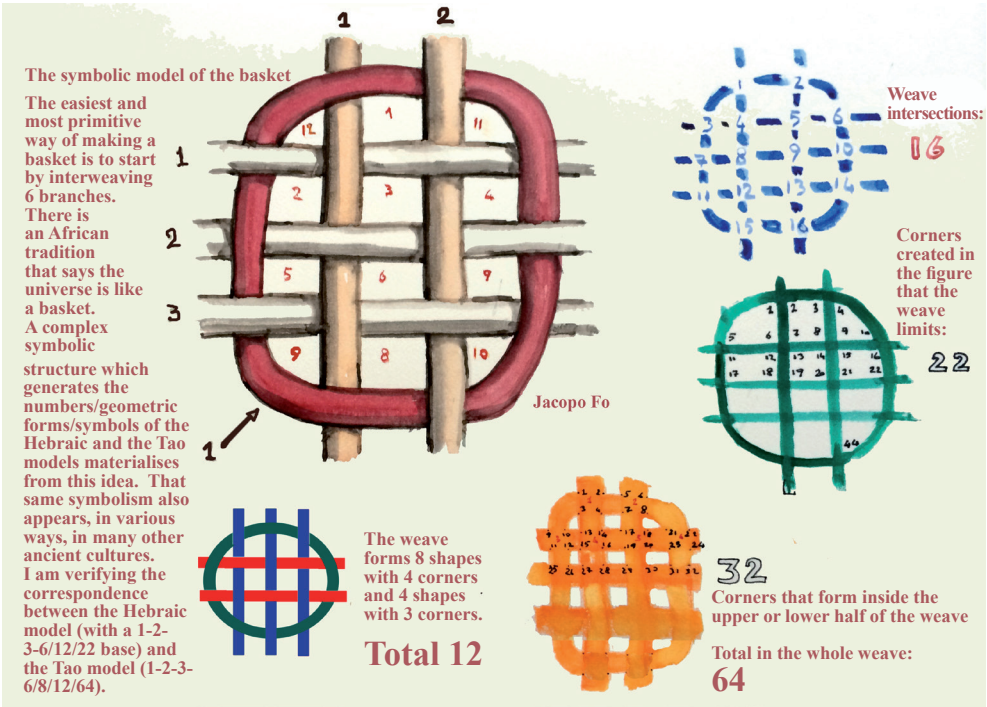
CHAPTER 10

The African origin of the Taoist system

To be honest, it has to be said that probably the Chinese mutated part of these ideas from African wise men.

In fact, the cosmogony of the Dogon people who, as already said, describe the universe as a basket shows considerable traces of this identification system.

The most elementary, and presumably the most primitive, way to start making a basket is to weave 2 branches with 3 branches and then block them by intertwining a sixth circular branch around the first 5 *(Figure 22)*.



(Figure 22)

In this manner you obtain a shape which, in addition to being superimposable on the already mentioned grid of equilateral triangles, defines 12 empty spaces inside it.

Still in Africa, but not only in the culture area of the Dogon people, a game called Mancala is very popular *(Figure 23)*, in-

(Figure 23)

volving a special board with 2 rows of 6 hollows in which a series of seeds or stones are placed, and which was used also as an abacus to count dozens.

Looking at it another way, we have in front of us a system for noting a binary code made up of 12 figures (2 hexagrams) using white and black stones.

It should be noted that according to some researchers, there were black Africans of the San ethnic group among the leaders of the Shang population, which conquered part of China in 1600 BC. According to these researchers, it was the San who spread the agricultural revolution through the world. Hypothetical proof of this comes from the fact that many words indicating agricultural processes contain the roots of san words.

Why did Pythagoras say that the universe was made of numbers? Why were the Jews convinced that God revealed himself in triangles?
Why did Taoists venerate hexagons?
Why did many ancient wise men from East to West have the same identical idea of the world?
They made the same discoveries because they followed the same elementary method: observing nature and reasoning on its forms.
They studied a discipline that Pythagoras called Arithmetic Geometry, and thanks to it they produced a model that described invisible particles of energy without material consistency and they discovered the constant ratios of proportion between numbers and the human body long before Fibonacci was born.
They invented Cartesian coordinates, fractals and binary numbering 2500 years before Liebeniz.
Leonardo da Vinci and Isaac Newton secretly studied their diagrams, but then were afraid of those ideas because they contrasted the academic knowledge of their times. The ideas seemed to be absurd: they were similar to some eccentricities of quantum physics.
This journey through the archaeology of thought stimulates a question, without mysticism: are the correspondences between the ancient diagrams and modern scientific discoveries just a strange coincidence, or is there something good in the extremely modest way used to observe the reality that accompanied the first steps of science?